TEACHER RESOURCE MANUAL

JUNIOR HIGH MATHEMATICS

RICULUM SUPPORT

Alberta
EDUCATION
1988

JUNIOR HIGH MATHEMATICS

TEACHER RESOURCE MANUAL

1988

Alberta
EDUCATION

NOTE: This publication is a support document. The advice and direction offered is suggested except where it duplicates or paraphrases the contents of the *Program of Studies for Junior High Schools*. In these instances, the content is screened in the same distinctive manner as this notice so that the reader may readily identify all prescriptive statements or segments of the document.

TABLE OF CONTENTS

GRADE 7 MATHEMATICS

GRADE 8 MATHEMATICS

GRADE 9 MATHEMATICS

APPENDICES

ACKNOWLEDGEMENTS

This manual has been prepared with guidance and input provided by a number of individuals. Their suggestions and assistance have proved to be valuable in the development of this document.

JUNIOR HIGH MATHEMATICS ADVISORY COMMITTEE

Mr. Pat McLaughlin (Chairman), Alberta Education
Ms. Linda Cox, Alberta Correspondence School
Mr. Don Darling, Edmonton School District #7
Dr. Tom Kieren, University of Alberta
Mr. Rene Mathieu, Alberta Education
Mr. Garry Canadine, Lethbridge School District #51
Ms. Teresa Scott, Grande Prairie School District #2357
Mr. Bruce Stonell, Stettler School District #1475
Mr. Glenn Wilson, County of Minburn #27

JUNIOR HIGH MATHEMATICS DEVELOPMENT COMMITTEE (Ad Hoc)

Mr. John Chapman, High Prairie School Division #48
Mrs. Jean Crawford, Calgary Roman Catholic Separate School District #1
Mrs. Myra Hood, Calgary School District #19
Mr. Ken Hutchings, County of Strathcona #20
Mr. Dick Kopan, Calgary School District #19
Mr. Peter Miller, County of Flagstaff #29
Mrs. Katie Pallos-Haden, County of Parkland #31
Mr. Leo Sam, County of Smoky Lake #13
Ms. Jan Sparling, Sturgeon School District #24
Ms. Susan Turner, Grande Prairie School District #2357

ADMINISTRATION AND PRODUCTION

This curriculum document has been prepared within the Junior High Unit, Curriculum Design Branch, Alberta Education.

Jim Brackenbury, Associate Director
Barbara Esdale, Learning Resources Officer
Alexandra Hildebrandt, Learning Resources Officer
Merv Lastiwka, Program Manager

Special recognition must go to the following staff members who helped to produce this document:

Lisa McCardle, Editor
Jan Cruickshank, Word Processing
Lin Gray, Word Processing
Denise Hugman, Word Processing
Cheryl Stoochnoff, Word Processing
Janice Yarmuch, Word Processing
Esther Yong, Word Processing

Abbreviations

The following abbreviations are used in this manual:

Hm	Holtmath
JM	Journeys in Math
PSCM	Problem-Solving Challenge for Mathematics
(R)	Remediation
(E)	Enrichment

PROGRAM RATIONALE AND PHILOSOPHY

Mathematics is an important component of education because it enables citizens to lead useful and productive lives and to be adaptive in an ever-changing technological society. The study of mathematics leads to a better understanding and appreciation of the quantitative and geometric nature of the concrete world and to the development of the knowledge, skills and positive attitudes necessary for decision making in personal living. All students should receive a level of mathematics education appropriate to their needs and abilities.

A mathematics program must provide a balance between a knowledge base and the application of that knowledge, especially in new situations and with new technologies. The pervasiveness of calculators and microcomputers and the increasing reliance of the economy on information transfer and processing have changed the ways in which mathematics is used in our society. The result is a substantial (and ongoing) change in emphasis within the familiar mathematical topics such as computational facility, problem solving, measurement and geometry.

The development of positive attitudes toward mathematics and learning is an essential element of a mathematics program in that it nurtures the confidence necessary for taking risks, accepting challenges and making decisions. Positive attitudes are generated by making mathematics meaningful and relevant to students, by selecting activities that are appropriate to students' abilities and by providing opportunities for students to experience success.

Each student must be viewed holistically and as capable of learning. Since self-concept influences learning and achievement, the program should encourage in each student a positive self-concept, and should focus on the growth of each individual. Appropriate and varying organizational and instructional strategies should be implemented to meet the diverse and individual needs of students.

Although junior high school students are at various stages of physical, emotional, and cognitive development, they all require experiences at a concrete level. Extensive experiences with concrete representations of mathematical concepts lead to intuitive understandings of abstractions. Students should be carefully guided from the concrete (model), through the transitional (pictorial representation) and eventually on to the formal (symbolic) level of cognition as mathematics concepts are being developed.

Junior high school students are in a transitional stage of life. Adolescence, characterized by rapid physical growth and the onset of puberty, is a period of uncertainty and great concern about peer relationships. The physical, intellectual, emotional, and social development of the students vary greatly. Supportive comments and guidance, and a genuine expression of concern for students, can help to maintain meaningful communication with students and enhance their learning.

The aim of the Junior High Mathematics Program is to develop an understanding of mathematics concepts by making mathematics relevant and concrete. The emphasis within the program must reflect the reality of the technological age. Appropriate experiences presented in a logical sequence will result in positive attitudes and positive learning outcomes.

GOALS AND OBJECTIVES

The goals of the Junior High Mathematics Program are to enable students to:

- solve problems and to grow in their capability to deal with new or different situations.
- use mathematics as a tool in the pursuit of personal goals and aspirations.
- develop a positive self-concept and a positive attitude toward mathematics and lifelong learning.

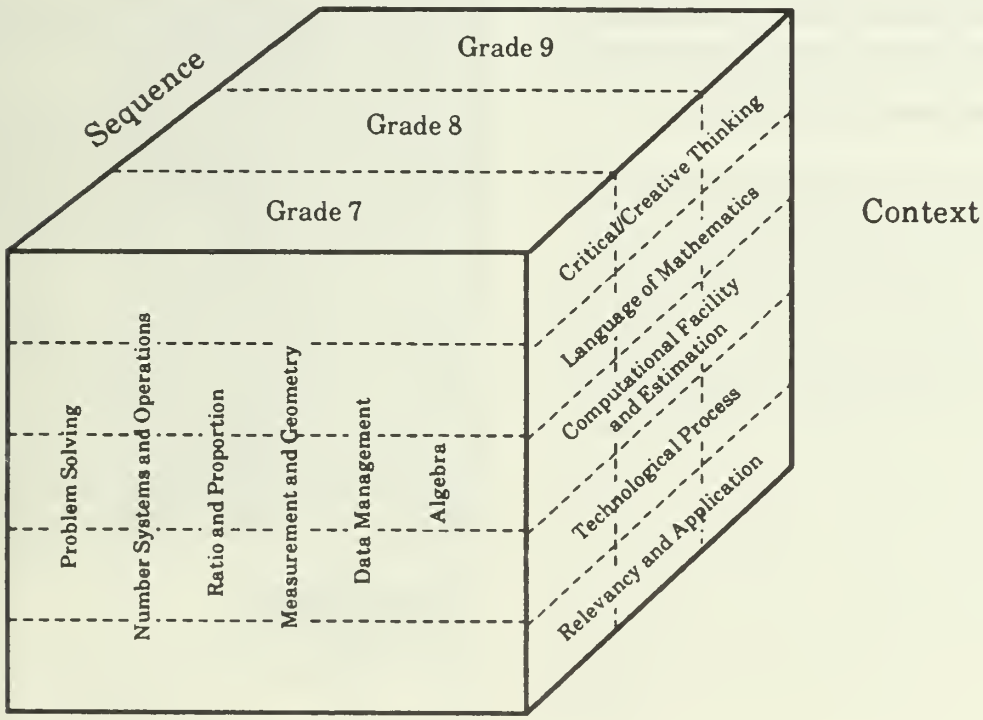
CONTENT

STRUCTURE OF THE PROGRAM

The content of the Junior High Mathematics Program is divided into six strands: problem solving; number systems and operations; ratio and proportion; measurement and geometry; data management; and algebra. The content is a consolidation of the skills and concepts developed in the elementary program and forms the basis for the further study of mathematics at the senior high school level. The skills and concepts within these strands are carefully sequenced over three grades taking into account the developmental nature of mathematics and the developmental nature of the learner. All students enrolled in this program should have an opportunity to complete it successfully.

There is an implicit dimension of the mathematics program that transcends the scope and sequence. It cannot be discretely taught as a unit of study nor can it be found in a chapter of a textbook. The context of the program is the element of teaching that creates and fosters positive attitudes, builds appropriate mindsets, and helps the learner interpret and understand the environment in relation to mathematics. Critical and creative thinking, the acquisition of quantitative concepts and skills (number sense), knowledge about and willingness to use technology, knowledge of the language and history of mathematics and the meaningfulness and relevancy of mathematics, must be modelled on a continuous basis and must be integrated into all strands of the program.

JUNIOR HIGH MATHEMATICS PROGRAM DIMENSIONS



Content Strands
(Basic Skills and Concepts)
(Scope)

The teacher can model and integrate these aspects of the mathematics program through his or her mediation or explanation to students. Understandings are learned, modified and refined over time, eventually building conceptions similar to what the teacher has in mind. The teacher observes students at a task and actively refines their understanding until the desired learning outcome is obtained. Teachers help students interpret these tasks by what they say about them (or by what they leave unsaid). For example, teachers who talk about the perplexing nature of problem solving are likely to impart to students the understanding that perplexity is a normal state in solving problems.

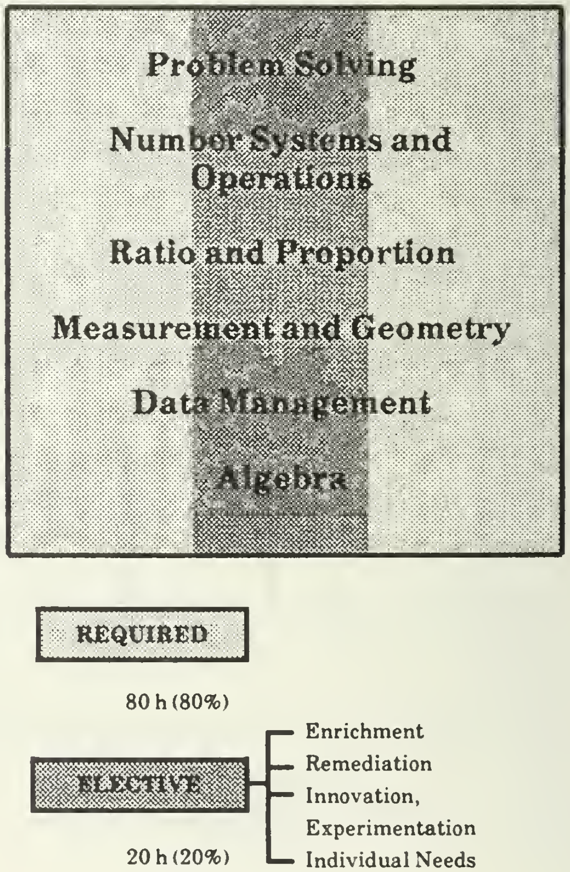
Student understandings and learning outcomes may not always be what teachers intend. For example, when students are always given busy work on computation, the understanding that they may develop is that "getting done" is more important than learning to compute. Students construct understandings about what is important, what to pay attention to and how to behave, from their own experiences and from tasks they encounter within the school experience. New experiences and tasks are combined with old understandings to build new understandings and conceptions.

REQUIRED-ELECTIVE FORMAT

The Junior High Mathematics Program has two components. The required component of the course outlined by the scope of the program describes the basic skills, knowledge and attitudes that all students should be expected to acquire. Of the 100-hour **minimum requirement** for the program, 80% (or 80 hours) shall be spent on this element of the course.

The elective component of the program shall be used to adapt and enhance the required portion of the course to meet the diverse and individual needs and capabilities of individual students. The activities associated with the elective must be integrated throughout the required component and shall be used to remediate and enrich student learning and/or to innovate and experiment with varying instructional and organizational strategies that may enhance student learning. The elective component is not intended to provide acceleration or advanced placement and therefore avoids unnecessary overlap with other courses or courses at a higher level. The maximum time allotment for the elective component shall be 20% of the instructional time.

In cases where the time allotted to the Junior High Mathematics Program exceeds the 100-hour minimum requirement, additional content may be presented to all the students. This content should extend and enhance the understanding of the knowledge, skills and attitudes in the required portion of the program.



SCOPE OF THE PROGRAM

Problem Solving

The most important goal of mathematics instruction is the development of students' ability to solve problems. The emphasis on problem solving requires a change in focus from exclusively finding answers to routine word problems to the acquisition and application of many different skills and strategies. Students should be able to apply these strategies to a variety of problem situations where the solutions are unknown and the means to the solution are not immediately evident.

Although problem solving is a legitimate goal in its own right, it should not be viewed as an isolated activity but, rather, as a group of related skills that are a part of a mathematics program. Because of the emphasis it must receive, problem solving appears both as a strand in the program and in an integrated form. The stages of problem solving and a variety of specific skills and strategies are identified and then developed within the strand. The skills, strategies and attitudes associated with problem solving are integrated into the rest of the program and should become part of the teaching philosophy.

Number Systems and Operations

Quantitative thinking and understanding and computational facility are still important goals of mathematics instruction. However, there must be a recognition that there are several ways to compute and today's students must be adept in all the methods. Students must be able to decide which method is most appropriate to the situation at hand and what degree of precision and accuracy is required.

Mental computation, paper-and-pencil operations, estimation and the use of calculators and/or microcomputers are computational strategies that must replace the singular emphasis on paper-and-pencil facility. Paper-and-pencil drills on arithmetic operations with more than three-digit numbers must be de-emphasized. Facility with one-digit number facts must be maintained. Activities that develop number sense and demonstrate the utility of mathematics in problem-solving situations shall increase in emphasis.

Working with numbers and number operations in a real world, problem-solving context gives meaning to numbers and to the operations with them. This is especially true of fractions and decimals. Emphasis shall be placed on the understanding of fractions and decimals as numbers and the comparisons of, and conversions between, fractions and decimals. Drill on operations of fractions with large denominators or multi-place decimals should be de-emphasized.

Mental computation involves finding natural and easy (not formal and algorithmic) strategies for calculations and results in an understanding of number relationships that cannot be replaced by technology. An understanding of the basic properties of number operations shall be developed for the purpose of doing mental calculations.

A heavy emphasis shall be placed on estimating measures and computations (including those that appear in complicated forms). Estimation requires a feel for numbers that goes beyond formal round-off procedures. Students must develop an estimation mindset that includes knowing what an estimate is, accepting its legitimacy, sensing when it is appropriate to estimate, recognizing how precise an estimate should be for a given situation and when a computed answer is sensible.

Ratio and Proportion

Ratio and proportion concepts, although they are an extension of the number systems and operations strand, have been collectively identified as a strand for the purpose of emphasis. The importance and use of equivalent representations in areas such as comparative shopping, scale drawings, model building, map reading, calculating wages, understanding and computing percents, and problem solving, as well as in the study of pure mathematics, cannot be over emphasized. A basic understanding of ratio and proportion must be developed at a concrete level. The applications of ratio and proportion, and percent are numerous and should be made meaningful and relevant to students.

Measurement and Geometry

SI metric measurement concepts and skills need to be consolidated in junior high school. Concrete experiences with making direct comparisons of objects with arbitrary units (e.g., the hand) and with standard units of length, area, volume, capacity and mass (e.g., cm, km^2, m^3, L, g) shall be provided. The need for large and small units of measure and the need to subdivide units into fractional parts should be emphasized. Formulas must be treated as useful tools for finding indirect measurements (e.g., speed) and for finding measurements indirectly (e.g., area). They shall be used after students understand the measure they are to calculate. Excessive memorization of formulas is discouraged.

Geometry is the study of the attributes and properties of various shapes and objects. Attributes to be considered are size and shape of one-, two-, and three-dimensional objects and the transformations of one- and two-dimensional shapes. The measurement of geometric attributes is best done in the context of measurement.

Data Management

People are confronted daily with data from which they must make personal and career decisions. Students must cope effectively with the vast amounts of data that they encounter. The importance of statistics, techniques for collecting and interpreting data, making predictions from data, and techniques for organizing and displaying data will constitute this strand.

Algebra

Algebra and algebraic thinking are not restricted to courses in the high school. From the time students enter school, they learn about generalizations in the form of symbolism, relations and functions. Open sentences ($\square + 2 = 8$) are used to express basic addition facts; ordered pairs are learned as a part of language development (associating a name with an object); relationships among numbers are learned through counting (less than, equal to, or greater than); and functions which have a unique ordered pair, given the first number, are used in learning basic number facts (e.g., in learning the three-times multiplication table, the set of answers 3, 6, 9... are a function of the counting numbers 1, 2, 3...). Graphs are pictorial representations of the relationship between unique pairs of numbers (e.g., heights of students plotted versus age of students).

The emphasis in this program is placed on the understanding of algebra as a generalization of the relationships and patterns in arithmetic. Evaluating expressions; solving equations; the development, interpretation and use of functions as they relate to formulas; and graphing linear functions, make up this strand.

THE ROLE OF CALCULATORS AND COMPUTERS

The rapid growth of microtechnology has had an immense impact on mathematics education. Standard computations and manipulations of algebraic symbols, for example, are now incidental applications of hand-held calculators. Mathematics programs must recognize the pervasiveness of technology by de-emphasizing activities that are much more easily replicated by computers, calculators and, in the future, by as yet unknown technologies. Emphasis must be placed on problem solving and on understanding concepts and relationships. Technologies such as computers and calculators must be used to develop concepts, to explore relationships, to explore patterns, to organize and display data, and to eliminate tedious computations.

AUTHORIZED LEARNING RESOURCES

DEFINITIONS

Learning resources fall into three categories: basic, recommended and supplementary. In terms of provincial policy, learning resources are those print, non-print and electronic software used by teachers or students to facilitate teaching and learning.

Basic learning resources are those learning resources approved by Alberta Education as the most appropriate for meeting the majority of goals and objectives of courses, or substantial components of courses outlined in provincial programs of studies.

AND

Those productivity software programs (e.g., word processors, spread sheets, data bases, integrated programs) approved by Alberta Education that can be used to achieve important objectives across two or more grade levels, subject areas or programs.

Recommended Learning resources are those learning resources approved by Alberta Education because they complement basic attainment of one or more of the major goals of courses outlined in the provincial programs of studies.

Supplementary learning resources are those learning resources approved by Alberta Education because they support courses outlined in the provincial programs of studies by enriching or reinforcing the learning experience.

BASIC and **RECOMMENDED** learning resources are available for purchase from the Learning Resources Distributing Centre.

AUTHORIZED LEARNING RESOURCES

The following learning resources support the junior high school mathematics program. The **abbreviations for the resources**, used throughout the curriculum guide, are noted in brackets.

A. Basic Learning Resources

Journeys in Math 7 (JM 7) by J.W. Lesage et al, Ginn (1987).

Journeys in Math 8 (JM 8) by R.D. Connely et al, Ginn (1987).

Journeys in Math 9 (JM 9) by W.C. Bober et al, Ginn (1988).

Holtmath 7 (Hm 7) by M.P. Bye et al, Holt, Rinehart & Winston (1984).

Holtmath 8 (Hm 8) by M.P. Bye et al, Holt, Rinehart & Winston (1984).

Holtmath 9 (Hm 9) by M.P. Bye et al, Holt, Rinehart & Winston (1986).

AppleWorks, Version 2.0, Apple Computer Inc. Note: This resource presently has been granted basic status for Grades 7-9.

B. Recommended Learning Resources

Journeys in Math 7, Teacher Resource Manual (TRM 7).

Journeys in Math 8, Teacher Resource Manual (TRM 8).

Journeys in Math 9, Teacher Resource Manual (still under development).

Holtmath 7, Teacher's Edition (TE 7).

Holtmath 8, Teacher's Edition (TE 8).

Holtmath 9, Teacher's Edition (TE 9).

Houghton Mifflin Mathematics 7, student text and teacher's resource book, by L. Dukowski et al, (1985).

Houghton Mifflin Mathematics 8, student text and teacher's resource book, by L. Dukowski et al, (1985).

Houghton Mifflin Mathematics 9, student text and teacher's resource book, by L. Dukowski et al, (1986).

Math Activities Courseware 7 (MAC 7), D. Super, Houghton Mifflin (1981).

Math Activities Courseware 8 (MAC 8), D. Super, Houghton Mifflin (1983).

C. **Supplementary Learning Resources**

Many supplementary computer courseware titles are listed in Alberta Education publications *Computer Courseware Evaluations: June 1985 to March 1986* (1986) and *Computer Courseware Evaluations: January 1987 to December 1987, Volume VII* (1988).

Other print and non-print resources will be announced as they are approved.

D. **Support Materials**

How to Develop Problem Solving Using a Calculator by J. Morris, National Council of Teachers of Mathematics (1981) (available from LRDC).

Problem Solving Challenge for Mathematics, (PSCM) Alberta Education (1985) (available from LRDC).

GRADE 7

PROBLEM SOLVING

7

OBJECTIVE 1. Demonstrates an understanding of a problem-solving situation.	Hm 7	JM 7
	11-13 224-225	28-30

CLARIFICATION OR EXAMPLE

Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time: 3 to 5 periods).

Brainstorm for a definition and examples. The following ideas should evolve about a problem:

a) it has no readily apparent solution or the means to the solution is not immediately evident

b) it can cause a person to be temporarily perplexed

c) it may have no answer, one answer, or more than one answer

d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature.

(See Journeys in Math 7 TRM, p. 30.)

ELECTIVE SUGGESTIONS

Problem-solving skills are essential for all students: being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success.

It is very important to recognize individual student differences in learning; therefore the growth expectations should also vary.

Individual needs can often be met by changing the conditions of a problem to make it simpler.

Manipulatives can also be used to meet individual needs.

e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.

2. Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

3. See: "Problem Solving Challenge for Mathematics" (Alberta Education, 1985) pp. 4-16.

OBJECTIVE 2. Demonstrates a willingness to find a solution to a problem.

CLARIFICATION OR EXAMPLE

In order to develop the students' willingness to find solutions, the teacher should:

- create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving
- be supportive and encourage risk taking in finding solutions
- encourage students to use creative approaches
- be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate)
- challenge students to think critically and justify strategies and solutions
- be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems
- provide appropriate questions for students
- present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life.

ELECTIVE SUGGESTIONS

Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.

e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?

Use of a concrete example will help students who experience difficulty with the operation of division.

Concrete approaches should be encouraged as long as it is necessary for the student.

A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:

a) other strategies and solutions

b) "what if?" (change an element of the problem)

c) generalization of rules to other situations.

e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that <u>no candies</u> are left over?"

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC and Sunburst Communications Software). (See "Computer Courseware Evaluations" Alberta Education for courseware reviews.)

The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.

OBJECTIVE 3. Uses a variety of strategies to solve problems.

CLARIFICATION OR EXAMPLE

To introduce the strategies of problem solving, an approach such as the following may be used: choose three non-related but similar problems that can be solved focussing on a strategy (consider that any problem usually requires the application of more than one strategy) such as the strategy of acting out or simulating the problem.

For students to become independent problem solvers, the first problem could be a teacher demonstration, the second could be a student trial with teacher guidance, and the third could be student practice.

The strategy of acting out or simulating the problem can be developed within the problem-solving framework [Understanding the problem, Developing a plan, Carrying out the plan, Looking back] as follows:

a) **Teacher Demonstration**

If six people were in a room and each one shook hands with every other person, how many handshakes were there? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 42, #29.)

b) **Teacher Guidance**

The students may now investigate a similar but non-related problem in a group situation by using the strategy of acting out or simulating. One such problem may be:

In how many ways can a committee of two be selected from five people?

c) **Student Practice**

A practice problem involving an acting out (or using manipulatives) strategy may be as follows:

Move three coins on the figure on the left to make it look like the figure on the right. ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 19, #7.7.)

The actual use of a manipulative could be very effective for lower ability students.

Evaluation

The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.

In assessing a student's problem-solving skills, the teacher should consider:

a) willingness to attempt problems

b) use of a systematic approach

c) selection of appropriate strategies

d) efficiency in selection of appropriate strategies

e) logical justification of strategies and solutions

f) perseverance

g) growth of confidence in problem-solving ability

h) transfer of problem-solving skills to situations other than mathematics.

Evaluation techniques and instruments for problem solving are suggested in "Problem Solving Challenge for Mathematics" (Alberta Education, 1985) pp. 7, 8, 52-56.

ELECTIVE SUGGESTIONS

Problem solving is integrated as an essential part of each strand. Enrichment/remediation and use of technological devices is therefore outlined in each strand.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 3: Uses a variety of strategies to solve problems.	Hm 7	JM 7	PSCM
The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:			50 (16,17)
a) Understanding the problem			
• knows the meaning of all the words in the problem			
• identifies key words		266, 386	
• draws a diagram	96, 97 240, 241	94-96, 220, 222	23 (7.17), 40 (14), 42 (36), 43 (6), 45 (14), 47 (7), 48 (16)
• classifies information as insufficient or extraneous		58, 59, 258-259 350	
• restates the problem in own words	254	196	
• uses concrete manipulatives	156, 157 352	69, 133, 139, 145, 301	19 (7.7), 20 (7.9), 41 (21), 47 (11), 48 (12,13)
• looks for a pattern	62, 136, 137, 149	160-162	19 (7.8), 20 (7.10), 40 (11), 50 (20)
• considers an alternative interpretation			

Hm 7 (Holtmath, Grade7)
JM 7 (Journeys in Math 7)
PSCM (Problem Solving Challenge for Mathematics)

	Hm 7	JM 7	PSCM
b) Developing a plan (choosing a strategy)			
• guesses and checks - improves the guess	41, 134, 135, 242, 280, 281	122-124 230, 272	26 (8.7), 27 (8.9), 39 (4), 40 (13), 41 (20), 41 (26), 43 (41), 44 (6), 49 (10)
• chooses and sequences mathematical operations		58-60 196-198	28 (8.10)
• acts out or simulates the problem	188, 189		42 (29,35), 45 (15)
• applies a pattern	62, 63 149	79	33 (9.6), 40 (11)
• uses a simpler problem	34, 35	196	21 (7.13)
c) Carrying out the plan • applies selected strategies • presents ideas clearly • documents the process • works with care • works in a group situation		28-30, 96, 124, 197, 198, 230, 258, 260, 286-288, 314, 320, 348, 349, 386	

d) Looking back	Hm 7	JM 7	PSCM
• determines if the answer is reasonable	6, 41	122, 215, 228, 229, 245	
• explains the answer in oral and written form			
• states the solution to the problem			
• restates the problem with the answer			
• considers other possible solutions to the problem		96, 313	
• looks for other ways to solve the problem		313	
• discusses solution process with others.			

NUMBER SYSTEMS

and

OPERATIONS

7

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 7	JM 7
	11, 12, 13	19, 49, 53, 58-60, 119, 169, 181, 197, 301

CLARIFICATION OR EXAMPLE

The intent of this objective, placed at the beginning of each strand, is to reinforce the fact that growth in students' ability to solve problems is a major goal of the program. Problem solving should not be viewed as an isolated activity but, rather, as a group of related activities, skills and attitudes that enhance students' capability to work in new or unfamiliar situation. A student's perplexity about a newly introduced concept or his/her inability to answer a question should be treated as a normal state in a problem-solving environment. The emphasis must be placed, not on finding a singular solution or strategy, but on the development of several strategies for understanding, or working towards a solution. The development of the knowledge, skills and attitudes associated with working in new or unfamiliar situations should become part of the teaching philosophy.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Students must be taught how to use calculator features after understanding of a concept is developed. Memory keys (order of operations) and percent keys operate on fundamental mathematics concepts. Students should be encouraged to discover these concepts. In addition, students must be taught how to interpret results. Knowing how to find a remainder or the repeating period of a rational number are implicit learning outcomes.

2. "Problem Solving Challenge for Mathematics" p. 41 Number 23.

OBJECTIVE 2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	Hm 7	JM 7
mental	15-19, 137, 141, 157	1, 9, 23, 31, 105, 177
estimation	6, 7,	8, 10, 16, 40, 46-48, 49, 50, 55
calculators	22, 28, 55, 69, 150, 182	17, 25, 42, 43, 55, 109, 111, 191

CLARIFICATION OR EXAMPLE

Computational facility should be developed and maintained throughout the school year.

An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged.

Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized.

Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve. Mental computational activities should occur on a regular basis.

Calculators should be used to develop understanding, to investigate patterns, and to perform tedious computations that do not enhance understanding.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 7, Program 10 "Number Patterns".

OBJECTIVE: A. Whole Numbers	Hm 7	JM 7
1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers).	1-5, 8, 9, 14-17 24-25	1-12, 14-17, 61, 126, 171, 193, 215, 303, 309, 316

CLARIFICATION OR EXAMPLE

1. Use a diagnostic tool to determine student proficiency with each of the operations.

2. Facility with basic facts through activities such as:

   a) Timed activities – e.g., Students may complete the same 80-100 basic facts questions each week. The challenge each week should be to improve scores and completion times, or both. While all four operations may be drilled, emphasis should be placed on addition and multiplication. Students can be taught to plot their results on a graph.

   b) Computer software – e.g., Math Activities Courseware (MAC) (Houghton-Mifflin).

   c) Engage students in frequent estimation in all classroom activities.

   d) Oral competitions – e.g., Use a "spelling-bee" model to drill students orally on basic facts.

ELECTIVE SUGGESTIONS

(R) To reinforce place value encourage the use of manipulatives. Each student should have his/her own material to handle. Make sure that proportional aids are used (these are aids that have a size difference).

e.g., These aids work effectively on the proportional level:

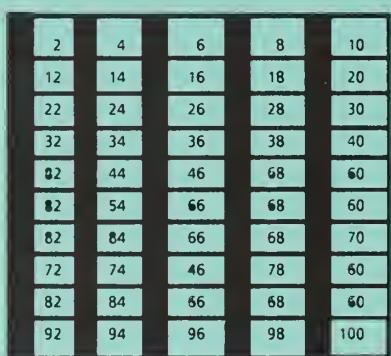
Exchange this	for this
10 beans	1 bean stick
10 white Cuisenaire rods	1 orange Cuisenaire rod
10 single beads	1 string of 10 beads
10 squares cut from graph paper	1, 10-square strip of graph paper

Work slowly with the aid. Once students understand the concept you can try using changeable aids. At this point you can move on to the concept at a more abstract level (adapted from Arithmetic Teacher, Vol. 32, #1, September 1984, p. 21).*

(E) Patterns in the Multiplication Tables.
Display a 10 x 10 hundred chart with stencils cut from tagboard which allow only one set of multiples to be displayed. Ask students what they see (Fig. 1). To allow all students to participate, have them begin quietly and independently. Have them write notes to themselves and then share ideas. Have students complete a 3s table (Fig. 2). Discuss the pattern that emerges. Ask students what happens when 2s and 3s are combined. (6s) (Continue doing the same activity with 4, 5, 6, 7, 8, 9.)

Fig. 1. Multiples of 2

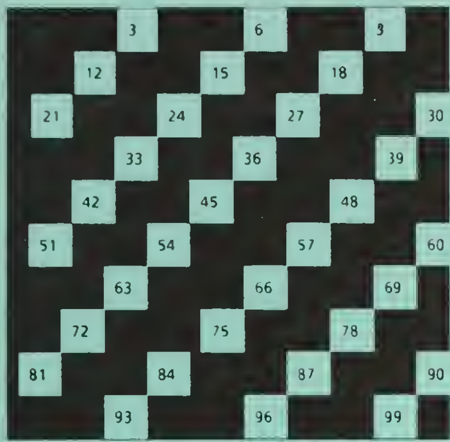
2s



* Reprinted with permission from National Council of Teachers of Mathematics.

Fig. 2. Multiples of 3

3s



(Arithmetic Teacher, Vol. 32, #7, March 1985, pp. 36-37)*

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Guess and check strategy can be useful when reinforcing skills/operations with whole numbers.

   e.g., Karen sold $50 worth of tickets. Student tickets were $2 and adult tickets were $3. If she sold 10 student tickets, how many adult tickets did she sell?
   a) Student guess.
   b) Ask questions for understanding of the problem: What are you finding? Price of tickets? Number of tickets sold? etc.
   c) Ask questions for solving (hints).
   d) Follow up – discuss/focus on the strategy students used – the solution and alternative solutions.

2. "Problem Solving Challenge for Mathematics" p. 45 Number 12; p. 50 Number 18.

OBJECTIVE: A. Whole Numbers
2. Understands properties of number operations and uses properties and relationships to perform mental computations (e.g., associative, commutative, distributive).

Hm 7	JM 7
6, 7, 8, 9, 15-19	22, 23

CLARIFICATION OR EXAMPLE

The intention here is to develop student understanding of the properties through their indirect use.

Encourage students to do many "mental exact" computations and then explain the properties.

---

* Reprinted with permission from National Council of Teachers of Mathematics.

e.g.,
1. $23 + 28 + 7 + 2 = (23 + 7) + (28 + 2)$
2. $16 + 9 + 4 = (16 + 4) + 9$
3. $2 \times 18 \times 10 = 36 \times 10$
4. $16 \times 12 = 10(12) + 6(12)$
5. add mentally from left to right

$$\begin{array}{r} \overrightarrow{236} \\ \underline{587} \end{array} \qquad \begin{array}{rcrcr} 200 & + & 500 & = & 700 \\ 30 & + & 80 & = & 110 \\ 6 & + & 7 & = & \underline{13} \\ & & & & 823 \end{array}$$

It is important that students understand the properties and relationships rather than define them. Students can demonstrate their understanding by their ability to verbalize.

ELECTIVE SUGGESTIONS

Timed drill activities on mental computations can be used to increase student skills.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: A. Whole Numbers 3. Understands that division by zero is undefined.	Hm 7	JM 7
	18-19	

CLARIFICATION OR EXAMPLE

Using a calculator or computer direct students to divide a given number by a set of numbers that become smaller (approach 0).

e.g.,
$$\begin{array}{rcrc} 100 & \div & 100 & = \\ 100 & \div & 50 & = \\ 100 & \div & 25 & = \\ 100 & \div & 20 & = \\ 100 & \div & 10 & = \\ 100 & \div & 5 & = \\ 100 & \div & 4 & = \\ 100 & \div & 2 & = \\ 100 & \div & 1 & = \\ 100 & \div & 0 & = \end{array}$$

Ask students to predict what would happen if they divided by 0. Check on a calculator. Does an "Error" message appear? Why?

ELECTIVE SUGGESTIONS

(R) What does division mean? (Repeated subtraction.) Use concrete manipulatives to count how many times, for example, a group of 2 can be subtracted from 6. Then ask, "How many times can 0 blocks be removed from 6?" Discussion should lead students to the conclusion that the question itself is meaningless.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: A. Whole Numbers
4. Writes the value of a power (whole number base and exponent).

Hm 7	JM 7
26-27	116

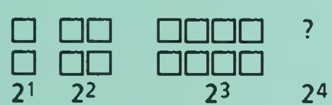
CLARIFICATION OR EXAMPLE

Use blocks to demonstrate/investigate patterns.

Teachers may wish to use alternative concrete manipulatives such as sticks, beans, coloured squares or grid paper.

e.g., #1:

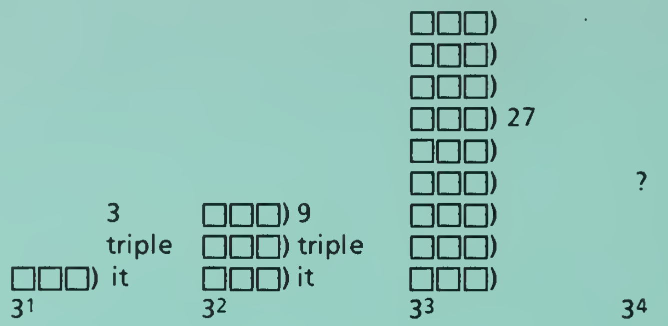
$2^1 =$ 2 (blocks)
$2^2 =$ 4 (blocks)
$2^3 =$ 8 (blocks)
$2^4 =$ 16 (blocks) etc.



Develop the concept further looking at the patterns with other numerical bases.

Explain to students that each result is doubling and therefore increasing the exponent by 1. By investigating patterns with 3s and 4s, students should realize that 3 or 4 is the factor and the number of times it has increased is the exponent.

e.g., #2:



Students can continue developing the concept to a factor of 10. They should be able to extend their understanding to include larger numerical factors.

ELECTIVE SUGGESTIONS

(E) Determine the last digit of the value of the power; e.g., 2^{27} (the last digit is 8).

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 8, Program 2A "Power Patterns".

OBJECTIVE: A. Whole Numbers 5. Applies the rules for the order of operations to evaluate expressions.	Hm 7	JM 7
	20-21	20-21, 25

CLARIFICATION OR EXAMPLE

The intent is to pose a problem involving the order of operations. Students should use calculators and computers to find a solution for $67 - 8 \times 3$.

As two solutions emerge, focus discussion on order of operations.

The following activity will reinforce this skill:

$(73 \square 26) \square 23 = 2277$

$(62 \square 21) \square 236 = 1066$

$1776 = (882 \square 49) \square 1758$

$215 \square 896 \square 788 \square 412 = 735$

Use the calculator to determine the missing signs. Encourage students to use mental estimation to help them.

ELECTIVE SUGGESTIONS

(E) Calculator Challenge students to develop a method which uses the memory function to solve problems using the order of operations.

e.g., 67 - 8x3
Start by clearing memory [MC]

Then 8x3 [=] [M +] 67 - [MR] [=] 43

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 4 "Calculator Quiz".
2. "Problem Solving Challenge for Mathematics" p.41 Number 18.

OBJECTIVE: A. Whole Numbers
6. Recognizes prime and composite numbers (limit: primes to 50).

Hm 7	JM 7
142, 143	112, 113, 118

CLARIFICATION OR EXAMPLE

Students should make all possible rectangles using a given prime number.

This concept is fully developed for factors and can be extended to prime numbers.

e.g., #1 :17


1
17

(only one arrangement possible)
(prime number)

e.g., #2:6


1
6

OR


2
3

(more than one arrangement possible)
(composite number)

ELECTIVE SUGGESTIONS

(R) Continue use of manipulatives:

e.g., Sieve of Eratosthenes.

A method of making a list of prime numbers was devised by an ancient Greek scholar, Eratosthenes, in the third century B.C. The method consists of taking a 100 chart and circling the first five prime numbers, 2, 3, 5, 7, and 11. Then all numbers divisible by 2, 3, 5, 7, and 11 are crossed out.
The prime numbers will not be crossed out.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(R) Prime Number Game.
Give students a list of the numbers from 1-50. Player 1 chooses a number (e.g., 15). This is crossed off the list. Player 2 strikes out all the factors of 15, (1, 3, 5) and adds them together to score 9 points. Player 1 again chooses another number and the same procedure continues until there are no numbers at the end of the game. Students will have to think carefully about the numbers they choose so they can get a higher point total.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

(E) This exercise can be completed using a utility program or with pencil computations.

1. 10 consecutive numbers with two or more digits – what is the maximum number of primes?
2. Set of 10 consecutive three-digit numbers with four? With only three? With only two? None? (Answer: 100-109; 130-139; 160-169; 110-119; 120-120.)
3. 11 and 101 are prime. Is 1 001? Check out 10 001, 100 001 and 1 000 001 on the computer.

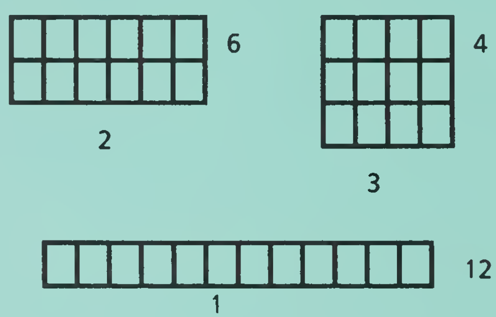
(Adapted from Arithmetic Teacher, Vol. 34, #5, January 1987.)*

OBJECTIVE: A. Whole Numbers 7. Lists the factors for whole numbers up to 200.	Hm	JM
	140-141	104-105, 108, 109

CLARIFICATION OR EXAMPLE

Develop a method for finding factors at a concrete and semi-concrete level before formal presentation.

e.g., Use tiles to develop factors of a given number. Using blank cardboard tiles, students should make all possible rectangles with an area of 12 square units. Draw and label each rectangle as it is formed.



The dimensions of the various rectangles reveal the factors of the given number (12).

* Reprinted with permission from National Council of Teachers of Mathematics.

ELECTIVE SUGGESTIONS

(E) Repeated division can be used to find factors of a number, as well as to reduce a fraction to lowest terms and give the L.C.M.

e.g.,
$$\begin{array}{r|rr} 2 & 24 & 36 \\ 2 & 12 & 18 \\ 3 & 6 & 9 \\ & 2 & 3 \end{array}$$

L.C.M. $= 2 \times 2 \times 3 \times 2 \times 3 = 72$

Reduce $\frac{24}{36}$

$$\begin{array}{r|rcr} 2 & 24 & / & 36 \\ 2 & 12 & / & 18 \\ 3 & 6 & / & 9 \\ & 2 & / & 3 \end{array}$$

$$\frac{24}{36} = \frac{2}{3}$$

Students may develop or use a simple program that generates a set of factors.

(Arithmetic Teacher, Vol. 34, #5, January 1987, p. 36.)*

<u>Basic Program</u>

```
100   PRINT "WHAT IS THE NUMBER";
110   INPUT N
120   PRINT "THE FACTORS ARE"
130   FOR K = 1 TO N
140   LET Q1 = N/K
150   LET Q2 = INT (N/K)
160   IF Q1 = Q2 THEN PRINT K
170   NEXT K
180   END
```

<u>LOGO Program</u>

```
T0    FACTOR :N
      MAKE "K 0
      PRINT (THE FACTORS ARE)
      CHECK :N :K
END
TO CHECK :N :K
REPEAT:N[MAKE "K:K + 1]
      TEST REMAINDER :N:K = 0
      IF TRUE THEN PRINT :K
END
```

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

See elective suggestions.

* Reprinted with permission from National Council of Teachers of Mathematics.

OBJECTIVE: A. Whole Numbers 8. Expresses a number as a product of its prime factors.	Hm 7	JM 7
	142, 143	114, 115

CLARIFICATION OR EXAMPLE

Encourage the use of calculators, factor trees and/or factor stacks.

Repeated division on a calculator or using the repeated division method can be helpful. Students must remember that only primes can be used.

With a calculator, guess and check can be used to see what primes go into a given number.

ELECTIVE SUGGESTIONS

(E) Have students look at numbers whose prime factorization consists of one prime factor; e.g., $16 = 2 \times 2 \times 2 \times 2$ OR 2^4.

All of the factors of 16 can be expressed as $1, 2^1, 2^2, 2^3$ and 2^4. What is the relationship? (Five factors → one more than the exponent.) Have students experiment with other numbers.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem Solving: At a junior high school there are 1000 students and 1000 lockers. The lockers are numbered in order from 1 to 1000. A student entered the building and opened every locker. A second student closed every locker having an even number. A third student changed every third locker, closing those that were open and opening those that were closed. A fourth student changed the fourth locker, and so on. This continued until all 1000 students passed through the locker room. What was the position of locker #1000?

Strategies: Reduce to a simpler problem; e.g., What if there were only 20 students and 20 lockers?

Make a chart to organize the data.

Students

Locker #

	1	2	3	4
1	0			
2	0	C		
3	0	0	C	
4	0	C		0

Extension: How many students changed the position of locker #600?

OBJECTIVE: A. Whole Numbers 9. Uses a calculator or microcomputer to generate a multiples of a given number.	Hm 7	JM 7
	151	111

CLARIFICATION OR EXAMPLE

Reinforce concept of multiples with a simple verbal activity.

e.g., count by 2's
count by 3's
count by 5's

Have them investigate ways they can generate sets of multiples on a calculator.

e.g., addition, multiplication.

Develop a program on the computer to generate multiples of a given number.

```
  5 REM MULTIPLES OF A NUMBER
 10 HOME
 20 INPUT "WHAT IS THE NUMBER THAT YOU WISH SEE THE MULTIPLES FOR?"; N
 30 INPUT "HOW MANY MULTIPLES DO YOU WANT LISTED?"; M
 40 HOME
 50 PRINT "MULTIPLES OF "; M
 60 PRINT " - - - - - - - - - - - - -"
 70 FOR X = 1 TO M
 80 PRINT N * X,
 90 NEXT
100 END
```

ELECTIVE SUGGESTIONS

(E) Use the computer program to generate common multiples for two or more numbers.

(R) Have students use utility programs to generate multiples.

This is a simple BASIC program that will find the LCM of three numbers:

```
100 REM FIND THE LEAST COMMON MULTIPLE
110 READ A, B, C
120 LET X = A
130 IF INT (X/A) = X/A THEN 160
140 LET X = X + 1
150 GO TO 130
160 IF INT (X/B) = X/B THEN 190
170 LET X = X + 1
180 GO TO 130
190 IF INT (X/C) = X/C THEN 220
200 LET X = X + 1
210 GO TO 130
220 PRINT "THELCM OF"; A; B; C; "IS"; X
230 GO TO 110
240 DATA
250 DATA
260 END
RUN
```

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Include in development of the lesson.

Problems

1. There are two sizes of tables in a banquet hall. One size seats exactly 5 people and the other size seats exactly 8 people. At tonight's banquet, exactly 79 people will be seated at less than one dozen tables, and there will be no empty places. How many tables of each size will there be?

2. The members of a flag squad wanted to arrange themselves into rows with exactly the same number of squad members in each row. They tried rows of 2, 3, and 4, but there was always one squad member missing. Finally they were able to arrange themselves into rows with exactly 5 in each row. What is the least number of members in the flag squad?

   (pp. 90-91, Creative Problem Solving, G. Lenchner, Houghton Mifflin Company.)*

OBJECTIVE: A. Whole Numbers 10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10.	Hm 7	JM 7
	138-139	106-107

CLARIFICATION OR EXAMPLE

Use the calculator to reinforce divisibility rules.

Teachers may also wish to review divisibility rules with the use of charts.

ELECTIVE SUGGESTIONS

(R) Make divisibility charts from a 10X10 hundreds chart (numbers from 1 to 100). Students can make posters to show which tables have ending rules or sum of digit rules or one that displays 2s, 5s, and 10s tables and another that displays 3s and 9s tables. Display the posters with questions underneath.

Fig. 4. Divisibility rules: Ending rules

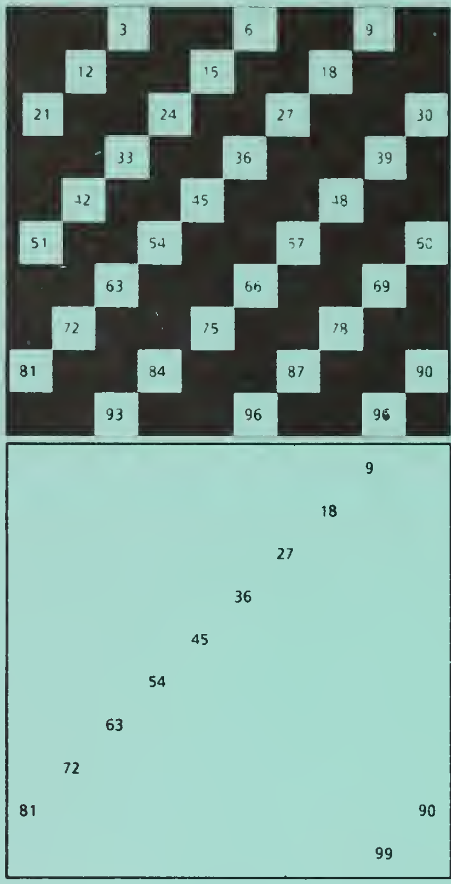
2	4	6	8	10
12	14	16	18	20
22	24	26	28	30
32	34	36	38	40
42	44	46	48	50
52	54	56	58	60
62	64	66	68	70
72	74	76	78	80
82	84	86	88	90
92	94	96	98	100

5s

5	10
15	20
25	30
35	40
45	50
55	60
65	70
75	80
85	90
95	100

a) What tables are these?
b) Look at the digits in the units place. Do you see a pattern?
c) How can you recognize the numbers that belong in these tables?
d) In each of these tables, what are the next five numbers after 100?
e) Our number system is base ten; in base ten the 2s, 5s, and 10s tables have ending rules.

Fig. 5. Divisibility rules: Sum of digits



What tables are these?
Add the digits in each number.
What pattern do you see in the answers?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Develop a computer program which tests for divisibility.

Problems

1. The pages of a certain book are numbered consecutively from 1 to 500. How many page numbers meet each of the following conditions?
   a) The page numbers contain the digit 5 and are divisible by 5.
   b) The page numbers contain the digit 5 but are not divisible by 5.
   c) The page numbers do not contain the digit 5 but are divisible by 5.

2. Replace the missing digit so that the resulting number is divisible by 9.
   a) 1 4 5 6 □ 28
   b) □ 6 49
   c) □ 54 321

   Replace the missing digit so that the resulting number is divisible by 3.

3. "Problem Solving Challenge for Mathematics" p. 24 Problem 8.2; p. 40 Number 16.

OBJECTIVE:	B. Decimals 1. **Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers).**	Hm 7	JM 7
		33, 36, 38-39, 42-44, 48-51	33-35 40-53

CLARIFICATION OR EXAMPLE

Assess student proficiency in basic operations (see Objective #1, whole numbers for drill and practice activities).

Encourage the use of varying computational strategies when reviewing basic operations.

(Game: Adapted from Arithmetic Teacher, Vol. 32, #6, February 1985, p.56.)*

Calculating by Teams

Objective: To practise computation with decimals, emphasizing speed and accuracy.
Directions:

1. On an $8\frac{1}{2}$" × 11" sheet of paper, create six questions in large writing like these (for decimals):
   a) $2.45 + 6.8 = \square$
   b) $\square - 0.027 = \bigcirc$
   c) $5.1 + 7 + \bigcirc = \Delta$
   d) $\Delta - 10.023 = ▭$
   e) $▭ + 6.01 = ⬭$
   f) $2.69 + ⬭ = \bigstar$

   Reproduce the sheets so that you have as many as the number of rows of six students.
2. Cut the activity sheets into strips containing one problem.
3. Distribute the strips in order (a-f) down the row. When each student has a problem strip, give the signal to begin. The student with problem strip b cannot do the problem until the number that goes into the $\square$ is passed on from the student with problem strip 'a'.
4. The student with problem strip f brings the "final answer" to the teacher for checking. Students are encouraged to double-check their work and can signal at any time that a different answer is being passed back because of an error. The first row to submit the correct final answer is awarded one point for each player.

Going further

1. Have students make up problem strips for future relays. The teacher will eventually have a nice supply for classes in future years.
2. Make up strips with only decimals, or combinations of operations. This activity can be adapted for younger students by creating strips with easier problems.

---

* Reprinted with permission from National Council of Teachers of Mathematics.

3. Alternate methods for passing out strips to keep the competition fair (e.g., vertical or horizontal rows, front to back or back to front). Post a cumulative record of points earned.

(From the file of Leona Burke Worth, Township of Ocean School District, Oakhurst, NJ 07755.)

Write the digits in the boxes to make the largest and the smallest answers. Use each digit only once.

	Digits	Largest Answer	Smallest Answer
1.	9, 3, 1, 6	□□.7 + □.□	□□.7 + □.□
2.	3, 4, 7, 3	6□.□ − □.□	6□.□ − □.□
3.	8, 4, 7, 0	3.4□□ − 2.□□	3.4□□ − 2.□□
4.	2, 8, 6, 1	4.□□ × □□	4.□□ × □□

(Arithmetic Teacher, Vol. 34, #7, March 1987, p. 31)*

ELECTIVE SUGGESTIONS

(R) Use base 10 blocks to reinforce place value basic concepts.

(R) Time drills and reviews can be used to increase student proficiency.

(E) Discuss division of decimal numerals using a calculator. Ask students how they express the remainder when it is given in decimal form. How many are left over? Students could explore repeating remainders.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 1A "Decimal Dynamite".
   MAC 7, Program 1B "The Last Move".
2. MAC 8, Program 2B "Closing In".
3. "Problem Solving Challenge for Mathematics" p. 22 Problem 7.14.

OBJECTIVE: B. Decimals
2. Compares and orders decimal numbers.

Hm 7	JM 7
37	36-37

CLARIFICATION OR EXAMPLE

Fold a strip of paper repeatedly to form a decimal number line, giving students a tool to order and compare simple decimals. e.g.,

			0.5			
	0.25		0.5		0.75	
0.125	0.250	0.375	0.500	0.625	0.750	0.875

* Reprinted with permission from National Council of Teachers of Mathematics.

Game

Materials: Decimals written on $8\frac{1}{2}$" x 11" sheets of paper (one number on each sheet).

Directions

1. Give a decimal number to each class member.
2. Divide the class into teams of three to five students each.
3. Each team must go to a separate part of the room and arrange its members in a line, with the member holding the smallest number in the front and other team members ranked behind in the order of the numbers held.
4. The first team to arrange itself in the correct order wins.
5. The team size can be increased to make the game more difficult.

This ability can be modified for lower grades to teach several concepts involving ordering.

ELECTIVE SUGGESTIONS

(E) Extend the folding experiment and have students address the concept of infinity between two points.

Teachers may wish to develop dot to dot games that order decimals from least to greatest.

(Arithmetic Teacher, Vol. 34, #7, March 1987.)*

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Develop or use a program to order/compare decimals.
2. MAC 7, Program 2 "Multi-Master".

OBJECTIVE: B. Decimals
3. Rounds decimal numbers.

Hm 7	JM 7
40-41	38-39

CLARIFICATION OR EXAMPLE

Give real life examples to demonstrate the need for rounding decimals.

e.g., If 1 litre of gas costs 43.9¢, could you buy 25 litres of gas and get exact change back?

* Reprinted with permission from National Council of Teachers of Mathematics.

CLARIFICATION OR EXAMPLE

Encourage estimation and the use of calculator to check answers.

Have students think of other examples in everyday situations where decimals are rounded.

ELECTIVE SUGGESTIONS

(R) Give students a problem in which they would have to round the answer to increase their understanding.

e.g., Plane flying from Gander to London. Halfway across the ocean the plane develops problems. Should the plane go back or continue the journey?

(E) See Journeys in Math 7 TRM, p. 39.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: C. Fractions
1. Maintains previously developed skills with fractions (concept of a fraction, need for fractional numbers, equivalent fractions, basic fractions) at a concrete level.

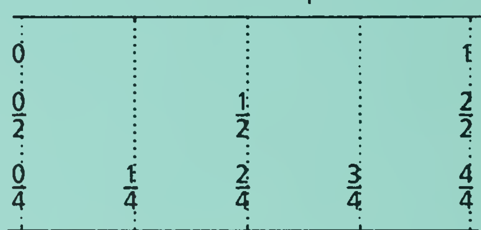
Hm 7	JM 7
155, 158, 159, 160, 161	167-171

CLARIFICATION OR EXAMPLE

Teachers may have students use strips of adding machine tape as a measuring unit and have them measure various objects and find the need to subdivide their unit, and discuss the concept of a fraction.

Students could fold their measuring unit into 2, 4, 8, 3, 6, etc., and label as directed.

Fraction Tape



Using this unit they should identify equivalent fractions.

Students may also use multiple boards to identify equivalent fractions.

e.g., $\frac{1}{3} = ?$

1	2	3	4	5	6	7	8	9	10	...
3	6	9	12	15	18	21	24	27	30	...

e.g., $\frac{1}{3} = \frac{3}{9}$, etc.

ELECTIVE SUGGESTIONS

(E) Develop fraction dominos to reinforce equivalent fractions.

	$\frac{4}{8}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{6}$	

(R) Continue to allow students to use fraction tapes/multiple boards until the concept is fully understood.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 7, Program 8A "Animal Farm".

OBJECTIVE: C. Fractions
2. Identifies mixed numbers and improper fractions and converts from one to the other.

Hm 7	JM 7
162, 164, 165	172-173

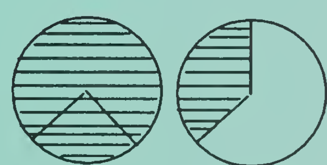
CLARIFICATION OR EXAMPLE

Students should develop this concept in pictorial form.
Divide 4 differents pizzas among 3 people.

Pizzas

Cheese | Bacon Pineapple | Pepperoni Mushroom | Mushroom Green Pepper

OR



Each person will receive $\frac{4}{3}$ or $1\frac{1}{3}$ pieces.

After some examples, students should develop the rules for conversion.

If students experience difficulty with this at the pictorial level it can be explained to them using concrete manipulatives; e.g., fraction tape. This concept should be developed until students can carry out the operation at an abstract level.

ELECTIVE SUGGESTIONS

(R) Encourage the use of manipulatives. Paper circles, pizzas, chocolate bars can be quite effective to illustrate this concept.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: C. Fractions
3. Orders fractional numbers.

Hm 7	JM 7
166-167	174-175

CLARIFICATION OR EXAMPLE

Use the measuring unit from Objective #1 to order fractional numbers (limit to those fractions shown). Use multiple boards to compare fractions.

e.g., compare $\frac{1}{2}$ and $\frac{4}{6}$

Form $\frac{1}{2}$ $\quad \frac{1}{2}\ \frac{2}{4}\ \frac{3}{6}\ \frac{4}{8}\ \frac{5}{10}\ \frac{6}{12}\ \frac{7}{14}\ \frac{8}{16}\ \frac{9}{18}\ \frac{10}{20}$

Form $\frac{4}{6}$ $\quad \frac{4}{6}\ \frac{8}{12}\ \frac{12}{18}\ \frac{16}{24}\ \frac{20}{30}\ \frac{24}{36}\ \frac{28}{42}\ \frac{32}{48}\ \frac{36}{54}\ \frac{40}{60}$

Since $\frac{3}{6} < \frac{4}{6}$

then $\frac{1}{2} < \frac{4}{6}$

ELECTIVE SUGGESTIONS

(E) Teachers may wish to use fractions with unusual denominators wherein the order is not as obvious.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Some students may write a utility program and others may use one to order fractions. This will also generate a group discussion on conversion from fractions to decimals.

2. MAC 7, Program 8A "Animal Farm".

OBJECTIVE: C. Fractions 4. Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators.	Hm 7	JM 7
	168, 169, 170, 171 (see T.E.)	176-180, 182, 185 (see T.R.M.)

CLARIFICATION OR EXAMPLE

While many students are able to use a "rules" approach to operating with fractions, there is much evidence to indicate that few students understand the operation.

A number of manipulatives may be used to demonstrate addition and subtraction.
Cuisenaire rods – Teachers and students should be familiar with the relationships among the various lengths and colours of the rods. (Instructions and activities are usually included when Cuisenaire rods are purchased.)

Fraction bars or fraction circles: these inexpensive cards can easily be constructed or found in commercial materials as black-line masters. Students should colour and cut out their circles, using a common colour for each fraction (e.g., thirds – green; quarters – blue etc.).

Adding fractions with common denominators (same colour) is straight forward, but students will be challenged to describe their answer when adding $\frac{1}{4} + \frac{1}{3}$ (1 blue + 1 green). A common denominator (common colour) must be found to describe the sum.

ELECTIVE SUGGESTIONS

(R) A multiplication table produces a multiple board for fractions, from which equivalent fractions can be determined.

e.g.,

1	2	3	4	5	6	...
2	4	6	8	10	12	...
3	6	9	12	15	18	...
4	8	12	16	...		

$$\frac{1}{2} + \frac{2}{3}$$

$$\frac{3}{6} + \frac{4}{6} = \frac{7}{6}$$

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 7, Program 8B "Fraction Challenge 1".

OBJECTIVE: C. Fractions 5. Writes number sentences to describe the addition and subtraction of fractions.	Hm 7	JM 7

CLARIFICATION OR EXAMPLE

The intention here is for students to demonstrate their understanding of the operation. It is not necessary for students to solve the question but to be able to write down what is being demonstrated with the concrete manipulatives.

This objective directly relates to the previous one and should be tied to the teaching of that objective.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: C. Fractions 6. Uses concrete manipulatives to demonstrate the multiplication and division of proper fractions.	Hm	JM
	174, 175, 178, 179	186-189

CLARIFICATION OR EXAMPLE

Multiplication

Illustrate this concept using a chocolate bar.

e.g., 

Eight friends share this chocolate bar equally. Shade the portion each person receives.

Bill decides to eat $\frac{1}{2}$ of his share. Shade the portion he eats.

tudents demonstrate their understanding by writing a number sentence.

$\frac{1}{2}$ of $\frac{1}{8}$ = $\frac{1}{16}$ (Discuss the meaning of the word "of".)

Division

Review the concept of division as repeated subtraction, then ask, "How many times can $\frac{1}{4}$ be subtracted from 1 $\frac{1}{4}$?"

tudents demonstrate their understanding by writing a number sentence.
See Journeys in Math 7 TRM, pp. 186-189.)

ELECTIVE SUGGESTIONS

R) Continue allowing students to use concrete manipulatives or pictorial form to develop full understanding.

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 9 "Fraction Challenge 11".

2. Problem

A farmer died leaving 17 cows. According to the terms of his will, the eldest child was to receive $\frac{1}{2}$ of the cows, the second child to receive $\frac{1}{3}$ and the youngest child to receive $\frac{1}{9}$. The children were puzzled about how to carry out the terms of their father's will since none of these fractional parts of 17 cows was a whole number. Finally, a generous neighbour offered to loan a cow to the children. They then had 18 cows: $\frac{1}{2}$ of 18 cows was 9 cows; $\frac{1}{3}$ of 18 was 6 cows; and $\frac{1}{9}$ of 18 was 2 cows. The total 9 + 6 + 2 was the original 17 cows. The 18th cow remained to be returned. How was this possible?

3. (See Journeys in Math 7 TRM, pp. 186-189.)

OBJECTIVE: C. Fractions 7. Writes number sentences to describe the multiplication and division of fractions.	Hm 7	JM 7

CLARIFICATION OR EXAMPLE

This objective should be taught with the previous objective. Students need only describe the concrete operations using number sentences, and should not be encouraged to compute at a formal level.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: D. Integers 1. Maintains previously developed skills with integers (concept of integers, need for integers, ordering of integers).	Hm 7	JM 7
	253, 256, 257, 258, 259, 260, 261, 262, 263	293-297

CLARIFICATION OR EXAMPLE

Demonstrate the need for integers. Use examples such as:

a) change in temperature
b) balancing chequebooks
c) above and below sea level
d) golf scores of above and below par
e) time zones.

ELECTIVE SUGGESTIONS

(R): Have students keep a temperature log for a week. Discuss daily changes; for example, Monday to Wednesday, or other variations.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 14 "Operation Integer".

2. Calculator Activity: Holtmath 7 Teacher's Edition, p. 257.

OBJECTIVE:	Hm 7	JM 7
OBJECTIVE: D. Integers **2. Uses concrete manipulatives to demonstrate the addition of integers.**	264, 266	298-300, 302-308

CLARIFICATION OR EXAMPLE

Concrete Activities: Journeys in Math 7 TRM pp. 292-303.

ELECTIVE SUGGESTIONS

(E) Journeys in Math 7 TRM, p. 299.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathmatics" p. 42 Number 37.

OBJECTIVE:	Hm 7	JM 7
OBJECTIVE: D. Integers **3. Writes number sentences to describe addition of integers.**	265	298-308

CLARIFICATION OR EXAMPLE

This objective should be combined with the previous objectives. Students should write number sentences to describe the concrete activity (adding integers). Adding integers at a formal level (development of addition rules) is discouraged.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

RATIO

and

PROPORTION

7

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.

Hm 7	JM 7
190, 191, 199, 195, 210, 211	205, 209, 212, 218, 224, 244, 245, 247, 230

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 49 Number 6.

OBJECTIVE 2. Maintains previously developed skills (identifies ratios as ordered pairs of numbers related to concrete situations; uses whole number constants to generate equivalent ratios).

Hm 7	JM 7
192, 193, 196, 197, 198	203-207, 216-217

CLARIFICATION OR EXAMPLE

Concrete Activities: Journeys in Math 7 TRM, pp. 204-207.
Holtmath 7 Teacher's Edition, p. 191 "Alternative Teaching Strategies".

ELECTIVE SUGGESTIONS

(R) Encourage the extended use of manipulatives for remediation.

(E) Have students develop their own parallel concrete activity.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 3. Uses concrete manipulatives to construct ratios in the following forms: a:b, a to b, and $\frac{a}{b}$	Hm 7	JM 7
	190-191 (see T.E.)	204, 205, 214, 216 (see T.R.M.)

CLARIFICATION OR EXAMPLE

The ratio $\frac{3}{6}$ can be expressed as 3:6 and 3 to 6.

Ensure that these forms are used interchangeably.

When evaluating the objective, present ratios in all three forms.

Holtmath 7 Teacher's Edition, p. 191.
Journeys in Math 7 TRM, p. 204.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Verifies the equivalence of two ratios using common multiples or factors: e.g., $\frac{14}{6} = \frac{7}{3}$ ($\div 2$, $\div 2$)	Hm 7	JM 7
	192, 193	204, 205, 206

CLARIFICATION OR EXAMPLE

Concrete Activities: Journeys in Math 7 TRM, p. 206.
Holtmath 7 Teacher's Edition, p. 193 "Alternative Teaching Strategies".

ELECTIVE SUGGESTIONS

(R) Students maintain facility at a concrete level.

See Holtmath 7 Teacher's Edition, p. 193.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 8, Program 9A "Ratio Rendezvous".

OBJECTIVE 5. Finds a missing element of a proportion using a common multiple of the elements:

e.g., $$\frac{3}{4} \xrightarrow{\times 3} = \frac{x}{12}$$ (×3 on both numerator and denominator)

Hm 7	JM 7
193, 195, 196, 198, 197	206, 207, 210, 211, 212, 213

CLARIFICATION OR EXAMPLE

The intent is to encourage the development of equivalence rules. (Do not allow the use of cross products.)

Concrete Activities: Journeys in Math 7 TRM, p. 210.

ELECTIVE SUGGESTIONS

(R) Students maintain facility at a concrete level (see #2).

(E & R) Journeys in Math 7 TRM, p. 211.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 17 Problem 7.4.

OBJECTIVE 6. Identifies percent as a ratio:

e.g., (p:100 or $\frac{p}{100}$)

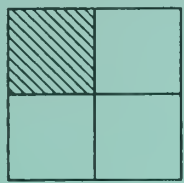
Hm 7	JM 7
204, 205	237

CLARIFICATION OR EXAMPLE

Use 10 x 10 cm grid paper. Have students calculate the total number of squares.

Shade a portion and write the ratio of the shaded portion to the whole. Write the percentage of the shaded portion to the whole.

Have students develop multiple examples to generate a rule for writing a percentage as a ratio.



10 cm

10 cm

e.g., Total 100 Shaded 25

Ratio $\frac{25}{100}$ Percent 25%

$\therefore \frac{25}{100}$ is 25%

ELECTIVE SUGGESTIONS

(E) Have students estimate percents on test scores. Have them check on their calculators.

(E) Explore the origin of the root word "cent".

e.g., <u>cent</u>ury

(R) Journeys in Math 7 TRM, pp. 236-237.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 7, Program 11 "Pie Graphics".

OBJECTIVE 7. Expresses ratios as percents and decimals and vice versa (limit: ratios in the form $\frac{a}{b}$, where b = 2, 4, 5, 10, 20, 25, 50)

e.g., $\frac{3}{4} \doteq \frac{75}{100} \doteq 75\% \doteq 0.75$

Hm 7	JM 7
204, 205	238, 239, 240, 241, 244

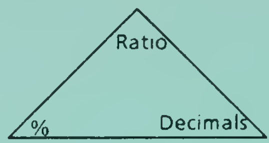
CLARIFICATION OR EXAMPLE

Emphasis should be placed on the notion that numbers have different equivalent forms.

Concrete Activities: Journeys in Math 7 TRM, p. 240.

The emphasis should be on simple ratios and mental conversions.

Drill cards will develop ability to convert from one to the other.



Cover one vertex and have student verbalize what is missing. Extend this by covering two vertices.

ELECTIVE SUGGESTIONS

(R) Journeys in Math 7 TRM, p. 241.

(Journeys in Math 7 TRM Teaching Aids Game 12, p. 76.)

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Students should use their calculators to find percents of their own test scores.

OBJECTIVE 8.	Hm 7	JM 7
Finds the percent of a number: e.g., 15% of 25	206, 207, 208, 209	242-243, 246, 247, 248, 249, 252-253

CLARIFICATION OR EXAMPLE

To give meaning to this objective, some background preparation is necessary. Students should demonstrate their understanding of percent in concrete terms.

e.g., Shade: 50% of a pie
100% of a pie
75% of a pie
25% of a pie
10% of a pie
1% of a pie

e.g., Money

50% of \$1.00 = 50¢

25% of \$1.00 = ______

How is the % of a number calculated? Emphasize the equivalent form of numbers: $50\% = \frac{1}{2} = 0.5$.

Then: 50% of $28 = 0.5 \times 28$ or $\frac{1}{2} \times 28$.

ELECTIVE SUGGESTIONS

(E) Journeys in Math 7 TRM, p. 243.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Estimate percentages of numbers. Check using calculators.
2. Holtmath 7, p. 210.
3. Journeys in Math p. 255.
4. Develop utility program to find percentages of numbers.
5. See: "Problem Solving Challenge for Mathematics" p. 16 Problem 7.1.

OBJECTIVE 9. Expresses one number as a percent of another number:
e.g., 12 is what percent of 16?
or $\frac{12}{16}$ = ____%?

Hm 7	JM 7
208, 209	239, 247, 251

CLARIFICATION OR EXAMPLE

Discuss in terms of real life examples. Where would the percent of a number be used?

e.g., test scores
sports results

Emphasis can be placed on estimation and the use of the calculator/computer to check the result.

e.g., 12 is what % of 16?

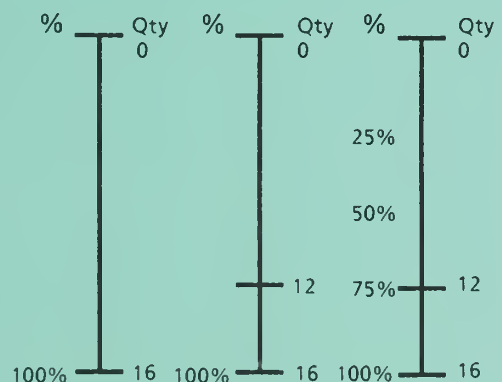


Fig. I – Place 16 opposite 100%

Fig. II – Estimate and position 12

Fig. III – Subdivide scale as necessary (50%, 25%)

ELECTIVE SUGGESTIONS

(E) The sale price of an article is $12. If the original cost was $16, what percentage was saved during the sale?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 8, Program 9B "Pulling Percents".

MEASUREMENT

and

GEOMETRY

7

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 7	JM 7
	62-63, 69, 73, 77, 83, 85, 91, 97, 331, 354, 355, 360	71, 83, 91

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing), be appraoched from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 34 Problem 9.8; p. 37 Problem 9.14.

OBJECTIVE 2. Maintains previously developed skills (concepts of linear, perimeter, area, volume, capacity and mass measures in concrete and pictorial forms; determines perimeter and area of right triangles and rectangles, and volumes of rectangular solids without formulas; uses protractor to determine the measure of an angle; transformational geometry).	Hm	JM
	61, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 100, 101, 329, 332, 333, 334, 335-342	65-71, 74-78, 80-85, 132, 133, 360, 361, 362, 363, 364, 365-379

CLARIFICATION OR EXAMPLE

Introduce concept of measurement. Use examples such as labels on food packages, speedometer numbers, sporting event distances.

Measurement should be taught as "a comparison to some arbitrary unit". These units are repeated and may be combined into larger units. The need to subdivide a unit should be demonstrated when a fraction of a unit is needed to describe length, mass, etc. Initially, the arbitrary unit may be one's hand. The need to standardize units (metres) should evolve.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 13 Problem 4; p. 16 Problem 7.2; p. 17 Problem 7.3; p. 18 Problem 7.6; p. 21 Problem 7.12; p. 44 Numbers 1, 3; p. 45 Number 11.

OBJECTIVE 2. Maintains previously developed skills
a) linear measure

CLARIFICATION OR EXAMPLE

Estimate and measure objects in the classroom such as the length of a paper clip, height of a door, or length of a piece of loose-leaf paper. Have students estimate and measure the length of given line segments. Have students draw line segments, estimating and measuring. When measuring, they can express the units (in m and cm).

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

<u>Problem</u>

Use maps and distance charts to solve problems. Students will need to interpret maps and charts, and plan using more than one possible path to a solution.

OBJECTIVE 2. Maintains previously developed skills
b) perimeter

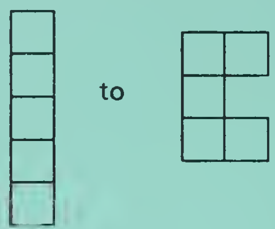
CLARIFICATION OR EXAMPLE

Estimate and measure the distance around various rectangular and triangular objects.

ELECTIVE SUGGESTIONS

(E) Explore how rearranging 5 square cards changes the perimeter.

e.g.,

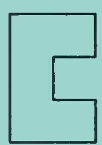


INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

<u>Problems</u>

1. Explore perimeters of compound figures.

e.g.,



2. "Problem Solving Challenge for Mathematics" p. 34 Problem 9.7.

OBJECTIVE 2. Maintains previously developed skills
c) area measure

CLARIFICATION OR EXAMPLE

Using a piece of ruled graph paper, cut out rectangles and triangles then determine their area by counting squares. Have students develop a strategy (such as a formula) for determining the area of rectangles or triangles.

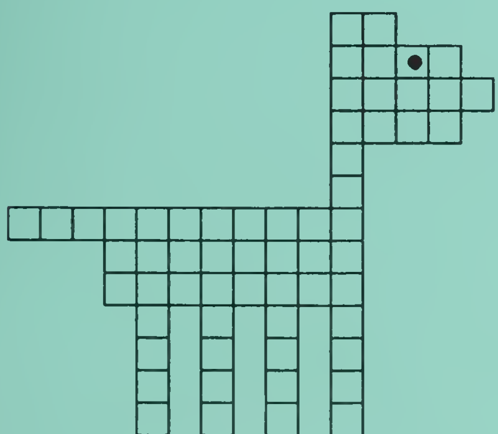
ELECTIVE SUGGESTIONS

1. MAC 7, Program 5 "Metric Mysteries".

2. (R) Create a Creature*

   Designate an area, such as 60 cm^2. Have the children cut out the area of squared paper into as many sections as they wish and then arrange the sections to form a creature. They will also enjoy creating creatures with the areas of certain parts specified. For example, they might use 24 cm^2 for the body, 16 cm^2 for the legs, and 20 cm^2 for other parts. Such a fanciful creature is shown below.

   A final project might be making creatures of any size and then calculating the area of the finished cutouts.



3. (E) Relating perimeter and area*

Instruct children to cut out as many rectangles and other polygons with a given area as they can. They should then figure the perimeter of each polygon and put the polygons in order from smallest to largest in perimeters. The class should share their results and list as many solutions to the problem as possible.

Next, have students cut out four rectangles, each having an area of 48 cm^2. They will realize that the shapes of these can vary. After they have cut out their four rectangles, invite them to cut each of them into two or more pieces to form another polygon. They can rearrange the pieces to form L-shaped figures (hexagons) and other polygons. Ask them to find and label the perimeter of each polygon. One-centimetre segments can be counted to find the perimeter for some polygons; for others in which the squared paper has not been cut at right angles, rulers can be used to find the lengths of the sides, or the perimeter of the polygons can be estimated.

(Arithmetic Teacher, Vol. 31, #4, December 1983, p. 11.)*

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Using a variety of tools such as string, grid paper, ruler, etc., have students try to determine the area of classroom objects such as a book front, desk top.

A square decimetre is a handy size for comparing with other objects. Have each child cut a square decimetre (10 cm × 10 cm) from squared paper. Specify several objects and ask them to compare the area of each to the square decimetre. They might use objects such as their textbook covers, a single surface of their crayon boxes, the area of their hands (outlined on paper), or the area of their desk tops. Lead them in a discussion of strategies for estimating area.

They might lay their square decimetres over an object whose area they want to estimate to see if the object's area is greater than, less than, or about equal to that of the square decimetre. They might see approximately how many times the decimetre square can be placed on a large area. They might also lay a smaller object on top of the square decimetre and estimate the fraction of the square decimetre that is taken up by the object.

OBJECTIVE 2. Maintains previously developed skills
d) volume measure

CLARIFICATION OR EXAMPLE

Have students construct rectangular prisms with centimetre cubes, and develop a strategy for determining the volume. Extend this to practical situations such as the volume of an aquarium.

* Reprinted with permission from National Council of Teachers of Mathematics

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem

Have students explore what happens to the volume of a cube if the length of a side is doubled. Use a calculator and prepare a chart.

OBJECTIVE 2. Maintains previously developed skills
e) capacity

CLARIFICATION OR EXAMPLE

In general, capacity units are used for liquids. The relationship of 1 mL = 1 cm^3 can be explored by using a waterproof rectangular solid, comparing the water it can hold to the cubic centimetre blocks it can hold.

Explore practical situations such as the capacity of a fuel tank, a pop bottle, a swimming pool, a grain bin.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 2. Maintains previously developed skills
f) mass

CLARIFICATION OR EXAMPLE

The mass of an object is "the amount" of material in that object. Two common mass units are the kilogram and gram. The feel of these can be found by estimating masses of objects by picking them up and then finding their actual mass on a balance. The appropriateness of other SI units (milligram, tonne) should be explored.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 2. Maintains previously developed skills
g) angle

CLARIFICATION OR EXAMPLE

Estimate angles, and then measure using a protractor. Some concrete examples are the hands of a clock, the corner on a table, the blades of scissors.

Discuss the inside and outside scale on the protractor.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem

How do you measure an angle whose rays are too small, or an angle that is greater than 180°?

OBJECTIVE: 2. Maintains previously developed skills
h) transformational geometry

CLARIFICATION OR EXAMPLE

Identifies and draws translations (slides), reflections (flips) and rotations (turns) of figures, and tests for congruence.

ELECTIVE SUGGESTIONS

(E) Explore tiling patterns (tessellations). Using shapes, try to tile a plane or design a tessellation with one or more shapes.

(Arithmetic Teacher, Vol. 31, #5, January 1984, p. 54.)*

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 8, Program 13 " Transformation".
2. Motion Geometry: "Floppy Math" Nelson Canada Ltd.

* Reprinted with permission from National Council of Teachers of Mathematics

OBJECTIVE: 3. Expresses equivalent measures of SI units (linear).	Hm 7	JM 7
	68, 69	66-67

CLARIFICATION OR EXAMPLE

The appropriateness of units should be explored. For example, the distance from the school to home should be measured in metres or kilometres but not centimetres. Conversions used in practical situations should be developed.

ELECTIVE SUGGESTIONS

(E) Explore through concrete situations the relationship of SI units of area, volume, and mass.

(R) Arrange students in groups. One group will draw lines of, for example, 5.2 cm, 6.5 cm, 7.8 cm, 15.0 cm. Another group will make line segments of 52 mm, 65 mm, 78 mm, 150 mm. Groups then try to find a match.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 4. Understands and uses the terms similar and congruent with respect to geometric figures.	Hm 7	JM 7
	332, 333, 335-342, 346	355, 134, 135, 380, 144, 146-147, 355

CLARIFICATION OR EXAMPLE

Congruence is having the same size and shape. Similar is having the same shape but not necessarily the same size (apply transformational geometry).

Students can be encouraged to draw congruent and similar shapes.

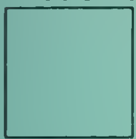
Journeys in Math 7 TRM, pp. 378-379.
Holtmath 7 Teacher's Edition, pp. 332-333.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 12 "Congruent FIgures".

2. Problem-Solving Activity:



Draw 2 lines so the picture shows 4 congruent triangles.

(Arithmetic Teacher, Vol. 31, #5, January 1984, p. 31.)*

OBJECTIVE:	Hm 7	JM 7
5. Understands and uses the term symmetry with respect to geometric shapes (line and turn symmetry).	127, 344, 348, 349, 350	129, 367, 376-377

CLARIFICATION OR EXAMPLE

Review lines of symmetry by dividing a given shape into two congruent parts by folding or by using a mira. Using a mira, draw the reflection in the mira and make a new shape with a line of symmetry. This can be reinforced by transformational geometry (flips).

The lid of a square box can be turned four different ways to fit the box. It has turn symmetry. The order of turn symmetry for a figure is the number of times it fits onto itself in one full turn. The order of turn symmetry for the square box is 4. Have students explore turn symmetry by tracing shapes and turning them to find their order of turn symmetry.

Lines of symmetry and turn symmetry can be reinforced by examining trademark designs of companies such as Shell Oil, Westinghouse Electric, Ralston Purina, Chrysler Corporation, or Mattel. A strategy to develop students' interest is to create an attractive bulletin board displaying examples of trademarks with line symmetry, turn symmetry, as well as combinations. Encourage students to explore examples that are not symmetrical.

ELECTIVE SUGGESTIONS

(E) Discuss graphic design by M.C. Escher. Produce patterns with turn symmetry.
Journeys in Math 7, pp. 374-375.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 6. Constructs geometric designs using tools such as a computer, compass, straightedge, ruler or mira.	Hm 7	JM 7
	121, 336, 341, 343, 357-359	134, 136, 137, 158-159, 148, 149, 151, 156, 158, 159

CLARIFICATION OR EXAMPLE

Some possible designs are company logos, tessellations of the plane (M.C. Escher). By exploring various designs, students learn the use of geometric tools and explore properties of geometric figures such as angles, size, shape, congruence and similarity.

Encourage students to be creative. One way is to arrange colourful and attractive bulletin board displays of geometric designs.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

A LOGO computer program may be used to create designs.

Journeys in Math 7 TRM, pp. 117, 120.

MAC 7, Program 6 "Little Logo".

DATA

MANAGEMENT

7

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 7	JM 7
	282, 300, 304	321, 327, 344, 349, 350

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Lessons can be designed to incorporate problem solving into data management, such as:

1. The student would ask questions in which statistical data would help to find an answer.
2. The student would decide what type of statistical measure would help to find the answer.
3. The student would collect and compute the necessary data.
4. The student would organize and interpret the data.
5. The student would answer the question.
6. The student would recognize the other choices of data and other interpretations, and should therefore be ready to defend or modify the conclusions.

OBJECTIVE: 2. Demonstrates a knowledge and understanding of the use and purposes of statistics as it affects daily living.	Hm 7	JM 7
	282, 283, 285, 300	338-341

CLARIFICATION OR EXAMPLE

The students can use newspapers to locate information such as weather reports, prices of used cars, sports statistics, and fashion prices. They should then display this information and find the high price, low price and "typical" price. The students should be able to answer the following questions:

1. How do numbers affect our daily lives?
2. What do the numbers compare?

3. List the possible sources of data:
   e.g., newspapers, magazines, books, radio, television, personal experiences, sports cards, opinion polls, local surveys, etc.
4. Why are statistics kept?
5. How do statistics help us?
6. How are statistics displayed clearly?

By relating to the students' environment, discuss how numbers are used in their daily lives. Use the above sources to explain what is being compared, why statistics are kept, how they help us, and how the results can be displayed (tables, graphs, etc).

ELECTIVE SUGGESTIONS

E) Investigate what a statistician is, and report what a career as a statistician would be like.

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 3. Collects and records data (tally sheets and frequency tables).	Hm 7	JM 7
	282, 283, 285	320-321

CLARIFICATION OR EXAMPLE

Use questions that require the students' output such as:

1. How many kilometres (or blocks) is your home from school?
2. What is your hair colour?
3. What is your eye colour?
4. What type of shoes are you wearing now?
5. What is your favourite sport?
6. Research data in the school library.

Use the tally method of fives (𝍸) to record the number of occurrences, and then write down the frequency counts.

The students answer questions concerning the greatest, smallest, and most typical occurrences. Use a month's temperature data for a town or city to find a way to organize the high and/or low temperatures into frequency tables, means, and ranges.

ELECTIVE SUGGESTIONS

(E) Using data from newspapers have the student make frequency tables, make up five questions on the data, exchange the data and questions with another student, and answer each other's questions.

(E) Use 49 cards (labelled 1 to 49) and play "Lotto 6-49" 20 times (draw 6 numbers each time), and record the results.

(Journeys in Math 7 TRM, p. 321.)

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE: 4. Understands and uses the term average (mean) as related to practical situations (e.g., test marks).	Hm 7	JM 7
	299, 301	336, 337

CLARIFICATION OR EXAMPLE

Use information such as bowling scores, test scores, students' weights, etc. The students will determine averages (means) by adding all the items (maximum of three digits per number) and dividing by the number of items.

Have groups of students list and compare items (e.g., number of hours of TV watching, ages in months, etc.).

ELECTIVE SUGGESTIONS

(R) Use smaller numbers and restrict sample size to determine mean.

(E) Find the missing mark in a list of marks when the mean is given.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Write a computer program that will calculate the mean for any list of numbers.

2. MAC 8, Program 1 "The Three M's".

3. Computer Average Program, Holtmath 7 Teacher's Edition, p. 301.

4. "Problem Solving Challenge for Mathematics" p. 42 Number 33.

Extended Content	Hm 7	JM 7
Find the mean from a frequency table.		

CLARIFICATION OR EXAMPLE

Use a list of students' weights to make a frequency table, and then use the frequency table to calculate the average.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 5.	Hm 7	JM 7
Maintains previously developed skills (Interprets data from pictographs, bar graphs, line graphs and circle graphs).	288- 290, 292-295	322-341

CLARIFICATION OR EXAMPLE

A graph displays and compares information. A pictograph uses symbols to display the data conveniently. A bar graph uses intervals and scales to display data. A line graph displays the relationship between two changing quantities. A circle graph represents data as being part of a whole.

Using graphs from any available sources, the students should be able to identify the types of graphs and to read information from each graph.

ELECTIVE SUGGESTIONS

Pictographs

(R) Practise skip counting by 25, 10 or whatever unit used in the pictograph.

Bar graphs and line graphs

(R) Use a square corner to align the horizontal and vertical scales, and check to see that students are reading the scales correctly.

Circle graphs

(R) Review measuring angles in combination with fraction circles.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs.

Hm 7	JM 7
298	322, 324, 326, 328, 330, 333-335

CLARIFICATION OR EXAMPLE

Each type of graph has its strengths and weaknesses. Pictographs are easy to read and appealing to look at but are not always very accurate. A bar graph is more accurate but is less appealing to look at. A line graph is best for displaying the relationship between two changing quantities, but has limits to its accuracy and care must be taken in making predictions based on trends.

A circle graph is good for displaying data as parts of the whole but can be more difficult to compare with other graphs.

The students can use data sources (such as newspapers, surveys or from previous exercises) to draw various types of graphs. The students should use knowledge of the strengths and weaknesses of each type of graph to select the form in which the data should be represented.

The students should be introduced to the steps in drawing a circle graph:

a) find the angle measure by multiplying each percent times 360°

b) construct a circle and use a protractor to draw each angle

c) label the graph.

ELECTIVE SUGGESTIONS

Pictographs

(E) Make pictographs that compare large numbers.

(R) Use topics which lend themselves to pictorial representation.

Bar graphs

(E) Discuss when to use horizontal bar graphs and when to use vertical bar graphs.

(E) Use scales that vary (don't start at 0 or are not constant).

Line graphs

(R) Use a line graph from a source to write a report describing what the graph is about and then reproduce the graph.

(E) Draw a double line graph (e.g., high/low temperatures, populations of Calgary and Edmonton over a period of years).

Circle graphs

(R) Review how to find percents of numbers.

(E) Given a circle graph with unlabelled sectors, have the students calculate what percent each sector represents.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 11 "Pie Graphics".

2. Use LOGO to construct graphs.

ALGEBRA

7

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 7	JM 7
	237, 240, 241, 254	286, 287, 288

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing), be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Understands and uses the term variable and uses variables to describe a concrete situation (e.g., number of jelly beans in a jar).	Hm 7	JM 7
	226, 227	266, 267

CLARIFICATION OR EXAMPLE

Use concrete situations to demonstrate the meaning and use of variables (a letter or symbol that represents an unknown number). For example, a variable can be used to express the number of pennies in a jar, or the age of a teacher.

ELECTIVE SUGGESTIONS

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Many BASIC and LOGO programs use variables in input statements. By varying the input value, the tudents can explore the effects. For example:

LOGO Program

```
TO SQUARE : SIDE
FD : SIDE
RT 90
FD : SIDE
RT 90
FD : SIDE
RT 90
FD : SIDE
RT 90
END
```

QUARE 30 draws a square with side length of 30, SQUARE 40 draws a square with side length 40, and so n.

BJECTIVE 3. **Uses variables to write mathematical expressions to represent practical situations (e.g., age of the students in the class in three years will be x + 3 years).**	Hm 7	JM 7
	226- 229	266- 269

CLARIFICATION OR EXAMPLE

Vith the inclusion of the operations +, −, × and ÷, we can represent practical situations using open nathematical expressions. For example, when three pennies are added to a jar containing an unknown umber of pennies, we can express this by $x + 3$.

Ise similar procedure for other simple expressions:

three less than a number: $x - 3$
5 times my age: $5 \times x$ or $5x$.

xtend the activity to more complex statements: three more than 5 times a number: $5x + 3$. Give tudents an expression such as $3a - 2$ and have them state its meaning in words.

ELECTIVE SUGGESTIONS

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Evaluates expressions for given values of the variable (limit: whole numbers, decimals).	Hm 7	JM 7
	232, 233, 234, 235	268, 269, 274

CLARIFICATION OR EXAMPLE

Expressions may be evaluated mentally, formally by substitution or organized in the form of tables.

Use practical situations to reinforce the use of mathematical expressions. For example, movie admission is $2.50. A table is one method of showing costs for different numbers of people.

Number	Cost
1	$2.50
2	$5.00
3	$7.50
p	$2.50 × p

The expression for cost is $2.50 × p or $2.50p.

From a table of values, students try to determine the defining expression by looking at the pattern.

x	
1	3
2	4
3	5

The defining expression is $x + 2$.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 1C "Guess and Test".

2. Problem

   For the normal range of summer temperatures the number of chirps made by a cricket in a minute is predictable. For example, at 16°C the cricket chirps 140 times per minute. At 24°C the cricket chirps 172 times per minute. What is the temperature at which the cricket chirps 180 times per minute? At 21°C, how many times will the cricket chirp per minute?

OBJECTIVE 5. Uses variables to write mathematical sentences to represent practical situations (e.g., people in a classroom = boys + girls + teachers or p = b + g + t).

Hm 7	JM 7
237, 238, 239	270, 271

CLARIFICATION OR EXAMPLE

With the basic operations (+ , −, x, ÷) and the inclusion of = , we can represent practical situations using closed mathematical sentences.

Mathematical sentences (equations) may have one unknown (i.e., a number plus 3 equals 7 becomes $x + 3 = 7$) or have more than one unknown (i.e., students in the room = boys + girls becomes $s = b + g$).

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

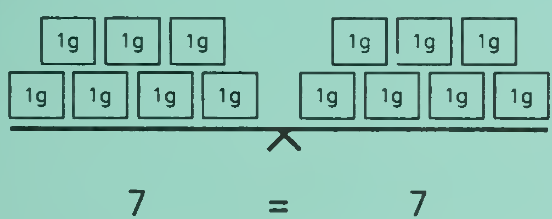
OBJECTIVE 6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality).

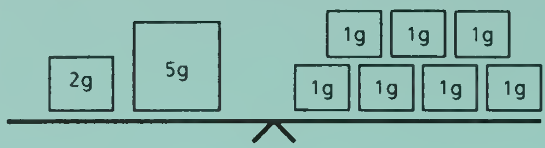
Hm 7	JM 7
	270, 271 (T.R.M.)

CLARIFICATION OR EXAMPLE

Using a balance scale and one gram masses, students should experiment to discover balance by manipulating the placement of the one gram masses.

Having discovered that an equal number of one gram masses is required on each side of the balance, students should replace the one gram masses on one side of the balance with different size masses but maintaining balance.





$$2 + 5 \quad = \quad 7$$

Thus students should discuss the concept of equality as a state of balance. For example:

$$7 = 7$$
$$2 + 5 = 7$$
$$3 + 4 = 7$$
$$1 + 6 = 7$$

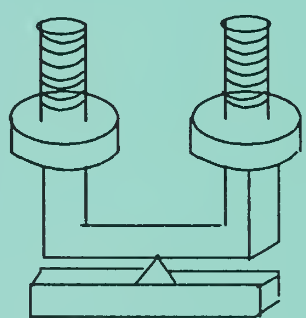
The forms used are limited to one unknown using whole numbers, positive fractions and positive decimals. Considerable time should be spent on the concept that an equation is a balance in which the left side equals the right side. A possible manipulative is a balance scale.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem

You have a pile of 24 coins. Twenty-three of these coins have the same weight and one is heavier than the others. Your task is to determine which coin is heavier. You are given a balance beam which will compare weights. Develop a strategy to find the heavier coin in a minimum number of weighings.



OBJECTIVE 7. Uses estimation, and guess and test procedures to solve equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$.	Hm 7	JM 7
	238, 239, 242	270, 271, 272, 273, 274

CLARIFICATION OR EXAMPLE

Solve	$2x + 3$	=	15 by guess and test
Try x = 5	$2 \times (5) + 3$	=	13 too small
Try x = 6	$2 \times (6) + 3$	=	15 ✓

This reinforces substitution of expressions and organizing work to document the process.

ELECTIVE SUGGESTIONS

(E) A simple classroom game is to have a student pick a secret number. Ask the student to perform some operation on that number (e.g., add 2) and then tell the class the result. Have class members find the secret number. This can be expanded to writing the equation and checking the solution. The game can be extended to include more than one operation and the sequence of the operations.

(R) "Roll It". Prepare cards with equations having whole number solutions between 2 and 12. For example,

$\boxed{x + 1 = 7}$ $\boxed{5b = 15}$

Deal 5 cards to each player. The first player rolls a pair of dice. Each player who has a card with an equation whose solution matches the dice discards that card. A discard may be challenged. If the discard was not correct, the player must pick up the discard and take one from the challenger. The first player with no cards wins.

(E) Journeys in Math 7 TRM, p. 277.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Use of calculators as a means of guess and test.

The automatic constant function is a valuable aid to guess and test.
e.g., To solve

$$\frac{x}{5} = 4.2$$

x = 10	$10 \div 5 = 2$ (low)
x = 15	$15 \div 5 = 3$ (low)
x = 20	$20 \div 5 = 4$ (low)
x = 23	$23 \div 5 = 4.6$ (high)
x = 21	$21 \div 5 = 4.2$ (✓)

OBJECTIVE 8. Verifies solutions to equations by substitution.	Hm 7	JM 7
	238, 239	276, 277, 278, 279, 282

CLARIFICATION OR EXAMPLE

Verification of a solution is simply trying to determine if the left side balances the right side.

Is 6 the solution to

$$\begin{aligned} 2x + 3 &= 13? \\ 2(\ ) + 3 &= 13? \\ 2(6) + 3 &= 13? \\ 12 + 3 &= 13? \\ 15 &\neq 13 \end{aligned}$$

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Encourage the use of a calculator. This reinforces the order of operations.

OBJECTIVE 9. Given ordered pairs, plot points on a coordinate plane.	Hm 7	JM 7
	316, 314, 315	319, 333, 330

CLARIFICATION OR EXAMPLE

THE GRAPHING COMPONENT OF THE ALGEBRA STRAND SHOULD BE DEVELOPED (AT ALL THREE GRADES LEVELS) WITH THE FOLLOWING CONTEXT IN MIND:

Numbers relate to each other. The emphasis in this program is placed on the understanding of algebra as a generalization of the relationships and patterns of pairs of numbers. Generally speaking, early elementary school mathematics deals with single numbers and their recognition and operations. The focus of secondary school mathematics shifts to 'pairs of numbers' and the relationships that exist between those pairs. Coordinate geometry (graphing), the use of formulas and eventually the study of trigonometric functions, polynomial functions, logarithmic functions, etc. become central to the secondary program.

Students should learn that the relationships occur naturally and are pervasive. They should feel comfortable in describing them. By the end of Grade 9, students should be able to identify the relations that are functions.

Relations are defined as the one-to-one correspondence that exists between two elements or two sets of data. Associating a name with an object, completing a 'times' or 'add' table, comparing the height to weight of people or comparing the area of a circle to its radius, are all relations.

Functions are those relations where the value of the first element determines the value of the second. Given the first element, only one possible pair of numbers can satisfy the condition of the function. The value of the second element "depends" on the value of the first element.

Example 1: When given the radius of a circle, only one value can describe the area. Area is a function of the radius.

Example 2: While the height of a person is related to weight, the relationship is **not** a functional one. A unique pair of numbers cannot describe the relationship (i.e., a person with a given height will not necessarily be a specific weight).

Graphs are pictorial representations of relations. Two types of skills are required for graphing. The first is the **technical skill** of choosing a scale, plotting points and drawing the lines (or curves). The second is the **interpretive skill** that contains elements such as increase, decrease, maximum, minimum, rate of change and slope. Because graphs contain a great deal of information in a relatively small amount of space, the interpretive skills are generally more difficult to deal with and are often neglected.

The development of graphing knowledge and skills are distributed as follows:

Grade 7 – technical skills (plotting points on a coordinate plane, choosing a scale, and drawing the graph).

Students should have plotted points in all four quadrants in Grade 6. In Grade 7 the emphasis should be placed on choosing appropriate scales and on investigating the effect of using different scales (e.g., distortion when different scales are used on each of the axes; accuracy of plots when large or small scales are used). Plotting a picture or design in all four quadrants is not only an interesting activity, but can effectively demonstrate the effect that varying scales can have on the outcome. It is important to continue to emphasize the notion of pairs of numbers and their relationships. (In this case the pairs of numbers will not have a functional relationship but, rather, serve to describe a location on a plane.)

Grade 8 – technical/interpretive skills (constructing a table of values that identify a function and graphing the relation).

Grade 9 – interpretive skills (given a graph or table of values, identify the function). (Limited to linear relations.)

ELECTIVE SUGGESTIONS

(E) Give each student a simple picture or design and have them transcribe it to a coordinate plane, then to identify the points needed to reproduce their picture.

Have students exchange with a classmate for checking by reproducing the picture from the given points.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Holtmath 7 Teacher's Edition, pp. 314-315.

GRADE 8

PROBLEM SOLVING

8

OBJECTIVE 1. Demonstrates an understanding of a problem-solving situation.	Hm 8	JM 8
	202, 203, 228	30-32, 64 98, 128, 162, 192, 302

CLARIFICATION OR EXAMPLE

Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time: 3 to 5 periods).

Brainstorm for a definition and examples. The following ideas should evolve about a problem:

a) it has no readily apparent solution or the means to the solution is not immediately evident

b) it can cause a person to be temporarily perplexed

c) it may have no answer, one answer, or more than one answer

d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature.

(See Journeys in Math 7 TRM, p. 30.)

ELECTIVE SUGGESTIONS

Problem-solving skills are essential for all students': being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success.

It is very important to recognize individual student differences in learning; therefore the growth expectations should also vary.

Individual needs can often be met by changing the conditions of a problem to make it simpler.

Manipulatives can also be used to meet individual needs.

e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.

2. Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

3. See: "Problem Solving Challenge for Mathematics" (Alberta Education, 1985) pp. 4-16.

OBJECTIVE 2. Demonstrates a willingness to find a solution to a problem.

CLARIFICATION OR EXAMPLE

In order to develop the students' willingness to find solutions, the teacher should:

a) create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving

b) be supportive and encourage risk taking in finding solutions

c) encourage students to use creative approaches

d) be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate)

e) challenge students to think critically and justify strategies and solutions

f) be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems

g) provide appropriate questions for students

h) present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life.

ELECTIVE SUGGESTIONS

Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.

e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?

Use of a concrete example will help students who experience difficulty with the operation of division.

Concrete approaches should be encouraged as long as it is necessary for the student.

A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:

a) other strategies and solutions

b) "what if?" (change an element of the problem)

c) generalization of rules to other situations.

e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that <u>no candies</u> are left over?"

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC and Sunburst Communications Software).

The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.

OBJECTIVE 3. Uses a variety of strategies to solve problems. Previously developed strategies are used.	Hm 8	JM 8	PSCM
	2, 38, 39, 132, 342, 343	126, 127, 160, 161, 190, 191, 268, 336, 370, 406	1-15

CLARIFICATION OR EXAMPLE

Students should encounter new situations that require an extension of problem-solving strategies acquired in Grade 7. The approach to this may be similar to that employed at the Grade 7 level whereby three non-related but similar problems can be chosen to focus on a particular strategy. Example: (Strategy – working backwards)

(1) Teacher Demonstration

The number of a past year is divided by 2 and the result turned upside down and divided by 3, then left right side up and divided by 2. Then the digits in the result are reversed to make 13. What is the past year? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 42, #28.)

(2) Teacher Guidance

A boy attempts to climb a 10 m pole. At every attempt he climbs 1 m and slips back 1/2 m. After how many attempts will he have reached the top? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 29, #8, 12.)

(3) Student Practice

Students will likely use the same strategy to answer the following questions.

Janice went to a store, spent half of her money, and then spent $10 more. She went to a second store, spent half of her remaining money, and then spent $10 more. Then she had no money left. How much money did she have in the beginning when she went to the first store? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 10, #2.)

<u>Evaluation</u>

The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.

In assessing a student's problem-solving skills, the teacher should consider:

a) willingness to attempt problems
b) use of systematic approach
c) selection of appropriate strategies
d) efficiency in selection of appropriate strategies
e) logical justification of strategies and solutions
f) perseverance

g) growth of confidence in problem-solving ability

h) transfer of problem-solving skills to situations other than mathematics.

Evaluation techniques and instruments for problem solving are suggested in "Problem Solving Challenge for Mathematics" (Alberta Education, 1985) pp. 7, 8, 52-56.

(See Holtmath 8 Teacher's Edition, p. iv Problem-Solving Strategies.)

ELECTIVE SUGGESTIONS

The Scope and Sequence in the Teacher's Edition (Holtmath 8) and Teacher Resource Manual (Journeys in Math 8) identify specific problem-solving strategies.

A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as:

1. other strategies and solutions
2. "what if?" (change an element of the problem)
3. generalization of rules to other situations.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Holtmath 8 Teacher's Edition, p. 83. "Using the Book".

e.g., Ann, Beth, Cathy, Dee, and Evita were playing in tennis matches to see who would play position 1, 2, 3, 4, or 5 on the girls' tennis team. Each girl played each of the other girls once. How many matches were played?

e.g., Children often use the *make-an-organized-list* strategy when solving this problem. First, a child might list all the matches that person A plays. Then the child would list all the matches played by persons B, C, D, and E. Note that once match AB is listed, match BA involves the same players and is not a different match.

AB	AC	AD	AE
BC	BD	BE	
CD	CE		
DE			

(Arithmetic Teacher, Vol. 32, #4, December 1984, p. 30.)*

* Reprinted with permission from National Council of Teachers of Mathematics.

BJECTIVE 3: Uses a variety of strategies to solve problems. Previously developed strategies are used.	Hm 8	JM 8	PSCM
The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:			
a) Understanding the problem			
• interprets pictures, charts and graphs			23 (7-16); 43 (42)
• asks relevant questions	67	190, 191	
b) Developing a plan (choosing a strategy)			
• collects and organizes information (charts and graphs)		62, 63, 226-228	20 (7-11); 31 (9.1); 39 (1); 40 (10); 42 (30-32, 34, 38); 49 (4,5, 8, 11); 50 (13, 14, 21)
• makes diagrams and models		96-97	
• experiments through the use of manipulatives		127	27 (8.8); 30 (8.14); 41 (22)
• breaks the problem into smaller parts			29 (8.13); 40 (12)
• works backward	102-103	334-335	18 (7.5); 29 (8.12); 39 (6); 42 (28); 45 (13); 48 (15)

Hm8 (Holtmath, Grade 8)
Jm 8 (Journeys in Math 8)
PSCM (Problem Solving in Mathematics)

	Hm 8	JM 8	PSCM
c) Carrying out the plan			
• applies selected strategies			
• presents ideas clearly			
• documents the process			
• works with care			
• works in a group situation			
d) Looking back			
• makes and solves similar problems	306-307		

NUMBER SYSTEMS

and

OPERATIONS

8

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 8	JM 8
	32, 33, 53	2, 3, 15, 21, 157-159 365

CLARIFICATION OR EXAMPLE

The intent of this objective, placed at the beginning of each strand, is to reinforce the fact that growth in students' ability to solve problems is a major goal of the program. Problem solving should not be viewed as an isolated activity but, rather, as a group of related activities, skills and attitudes that enhance students' capability to work in new or unfamiliar situations. A students' perplexity about a newly introduced concept or his/her inability to answer a question should be treated as a normal state in a problem solving environment. The emphasis must be placed, not on finding a singular solution or strategy, but on the development of several strategies for understanding, or working towards a solution. The development of the knowledge, skills and attitudes associated with working in new or unfamiliar situations should become part of the teaching philosophy.

ELECTIVE SUGGESTIONS

Develop "Happy Numbers" (from "Mathematics Teacher", p. 618, November 1986).*

(R) Find and display all happy numbers less than 100.

19 is a happy number. Let's see why.

$$19 \rightarrow 1 \times 1 + 9 \times 9 = 82$$
$$82 \rightarrow 8 \times 8 + 2 \times 2 = 68$$
$$68 \rightarrow 6 \times 6 + 8 \times 8 = 100$$
$$100 \rightarrow 1 \times 1 + 0 \times 0 + 0 \times 0 = 1$$

Since the sum of the squares of the digits of the generated sequence 19, 82, 69 and 100, ends with the number 1, we say that the number 19 is a happy number.

HAPPY NUMBERS

1	7	10	13	19
23	28	31	32	44
49	68	70	79	82
86	91	94	97	

Note: (E) = Enrichment
(R) = Remediation

* Reprinted with permission from National Council of Teachers of Mathematics

(E) Develop a program for "Happy Numbers", such as:

```
10  PRINT "HAPPY NUMBERS"
20  PRINT
30  REM CHECK ALL NATURAL NUMBERS LESS THAN 100
40  FOR X = 1 TO 99
50  Z = X
60  REM GENERATE SIX TERMS
70  FOR N = 1 TO 6
80  IF Z > = 100 THEN 170
90  IF Z > = 10 THEN 200
100 Z = Z * Z
110 NEXT N
120 REM CHECK FOR HAPPY NUMBERS
130 IF Z = 1 THEN PRINT X,
140 NEXT X
150 GOTO 220
160 REM ISOLATE THE DIGITS OF THE 3-DIGIT NUMBERS
170 H = INT (Z/100):T = INT (Z/10) - H * 10:U = Z - (H * 100 + T * 10)
180 Z = H * H + T * T + U * U: GOTO 110
190 REM ISOLATE THE DIGITS OF THE 2-DIGIT NUMBERS
200 T = INT (Z/10):U = Z - T * 10
210 Z = T * T + U * U: GOTO 110
220 END
```

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 28 Problem 8.11; p. 31 Problem 9.2.

OBJECTIVE 2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	Hm 8	JM 8
	4, 5, 34, 43, 59, 123, 126, 127, 217, 280, 281	4, 5, 24-25, 27, 48-49, 59, 83, 113, 145, 187, 285, 297, 401

CLARIFICATION OR EXAMPLE

An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged.

Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized.

Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve.

Calculators should be used to develop understanding to investigate patterns, and to perform (tedious) computations that do not enhance understanding.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. The following BASIC program should give your students an opportunity to practise their skills at estimation. The program can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals.

   Each player takes turns inputting a value of their choice. Then each is asked to estimate the results of using a particular operation on the two numbers. The estimate closest to the actual answer is declared the winner. This program is written in a simple form of BASIC and should run on any computer that uses BASIC.

```
  4 PRINT "ESTIMATION GAME"
 10 LET OP$ = "SUM"
 15 PRINT "IN TURN, EACH PLAYER WILL ENTER"
 20 PRINT "A NUMBER OF THEIR CHOICE"
 30 PRINT "THEN EACH PLAYER WILL ENTER AN"
 35 PRINT "ESTIMATE THE "OP$" OF THE NUMBERS"
 45 PRINT "THE CLOSEST ESTIMATE WINS"
 55 PRINT "FIRST PLAYER'S NUMBER" : INPUT A
 60 PRINT "SECOND PLAYER'S NUMBER" : INPUT B
 70 PRINT "FIRST PLAYER'S ESTIMATE. . .": INPUT A1
 80 PRINT "SECOND PLAYER'S ESTIMATE. . .": INPUT B1
 85 LET C = A + B: REM FINDS THE SUM
 90 IF ABS (C-A1) < ABS (C-B1) THEN 115
 96 IF ABS (C-A1) = ABS (C-B1) THEN 117
100 PRINT "PLAYER #2 WINS!"
105 GOTO 130
115 PRINT "PLAYER #1 WINS!"
116 GOTO 130
117 PRINT "IT'S A TIE!"
130 PRINT "PLAY AGAIN? (Y OR N)
131 INPUT ANS$
132 IF ANS$ = "Y" THEN GOTO 55
133 IF ANS$ < > "N" THEN GOTO 130
140 END
```

   Line 10 can be changed to indicate a different operation. By changing line 10 to LET OP$ = "PRODUCT", the instructions in line 35 tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also change. If OP$ = "PRODUCT", then line 85 must be LET C = A*B.

2. "Problem Solving Challenge for Mathematics" p. 42 Number 40.

OBJECTIVE A. Whole Numbers	Hm 8	JM 8
1. **Maintains previously developed skills with whole numbers (operations, order of operations, evaluation of expressions, prime numbers, factorization, divisibility).**	1, 3, 6-13, 22-25	1, 6-13, 22, 23, 27

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE A. Whole Numbers	Hm 8	JM 8
2. **Finds the greatest common factor.**	26, 27	134, 135

CLARIFICATION OR EXAMPLE

Review Grade 7 notes on whole numbers Objectives #7, 8 and 10 as methods to determine the GCF.

Use calculators and computers to determine the GCF of a set of numbers.

Methods of finding GCF's include:

1. listing the factors of the numbers, and finding the common factors.

   Example:
   Factors of 24 : {1, 2, 3, 4, 6, 8, 12, 24}
   Factors of 32 : {1, 2, 4, 8, 16, 32}
   GCF = 8

2. finding the prime factors of the numbers and then determining which factors are common.
   Example:

   $24 = 2 \times 2 \times 2 \times 3$
   $32 = 2 \times 2 \times 2 \times 2 \times 2$
   GCF $= 2 \times 2 \times 2 = 8$

Simplify: $\frac{24}{32}$

$$\frac{24 \div 8}{32 \div 8} = \frac{3}{4} \text{ (simplest terms)}$$

ELECTIVE SUGGESTIONS

The listings on the right can be used to find GCF's on a computer. The first listing will determine the GCF of two numbers. Students could type the program onto the computer and use it.

(E) An enrichment activity could be to have students either write their own program or improve the first listing.

(E) Another enrichment activity could be to change the first listing so that the computer will determine the GCF of more than two numbers. Listing (b) is a modification of listing (a); the changes and additions will allow the computer to find the GCF of four numbers.

(E) Use GCF to reduce fractions to lowest terms.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Darren wants to cut a 32 cm by 20 cm birthday cake into square pieces. What is the largest size he can cut? How many square pieces does he cut?

2. MAC 8, Program 6 "Number Stumper".

3. "Problem Solving Challenge for Mathematics" p. 32 Problem 9.4; p. 33 Problem 9.5; p. 50 Number 19.

BASIC Listing:

GCF: Two Numbers

```
5 REM GCF FOR 2 NUMBERS
10 HOME
20 INPUT "FIRST NUMBER?";A
30 INPUT "SECOND NUMBER?";B
40 IF A>B THEN N = B:GOTO 60
50 N = A
60 FOR X = 1 TO N
70 Y = A/X : Z = B/X
80 IF Y = INT(Y) AND Z = INT(Z) THEN CF = X
90 NEXT
100 PRINT:PRINT
110 PRINT "THE GREATEST
    COMMON FACTOR IS";CF
115 PRINT "AGAIN? (Y/N)"
120 INPUT ANS$
130 IF ANS$ = "Y" THEN GOTO 10
140 IF ANS$ < > "N" THEN GOTO 115
150 END
```

GCF: Four Numbers

```
5 REM GCF FOR 4 NUMBERS
10 HOME
20 INPUT "FIRST NUMBER?";A
30 N = A
40 INPUT "SECOND NUMBER?";B
50 IF A>B THEN N = B
60 INPUT "THIRD NUMBER?";C
70 IF C<N THEN N = C
80 INPUT "FOURTH NUMBER?";D
90 IF D<N THEN N = D
100 FOR X = 1 TO N
110 Y1 = A/X:Y2 = B/X:Y3 = C/X: Y4 = D/X
120 IF Y1 = INT (Y1) AND Y2 = INT(Y2) AND
    Y3 = INT(Y3) AND Y4 = INT(Y4) THEN CF = X
130 NEXT
140 PRINT : PRINT
150 PRINT "THE GREATEST COMMON FACTOR
    IS:";CF
160 PRINT "AGAIN? (Y/N)"
170 INPUT ANS$
180 IF ANS$ = "Y" THEN GOTO 10
190 IF ANS$ < > "N" THEN GOTO 160
200 END
```

OBJECTIVE A. Whole Numbers

3. Finds the lowest common multiple.

Hm 8	JM 8
28, 29	134, 135

CLARIFICATION OR EXAMPLE

Apply concepts from Grade 7 Whole Numbers Objectives #7, 8, and 10 as strategies to attain the LCM.

Try a guess and check strategy to find LCM.

Relate LCM to lowest common denominator to assist in making equivalent fractions for operational purposes. Stress the meanings rather than the acronym (GCF, LCM).

ELECTIVE SUGGESTIONS

(E) Program a computer to <u>select</u> the LCM of given numbers. (See previous objective.)

(E) Challenge students to improve or change the BASIC computer program below. Write a program to find the LCM of more than two numbers.

(E) Utilize the relationship between the GCF and LCM of two numbers (A and B): GCF × LCM = A × B.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. BASIC program to find the LCM of two numbers:

```
5   REM LCM OF 2 NUMBERS
10  HOME
20  INPUT "FIRST NUMBER?";A
30  INPUT "SECOND NUMBER?";B
40  IF A<B THEN N = B:P = A:GOTO 60
50  N = A: P = B
60  X = 0
70  X = X + 1
80  Y = N * X/P
90  IF Y = INT(Y) THEN LCM = N*X: GOTO 110
100 GOTO 70
110 PRINT:PRINT
120 PRINT "THE LOWEST COMMON MULTIPLE IS"; LCM
130 PRINT "AGAIN? (Y/N)"
140 INPUT ANS$
150 IF ANS$ = "Y" THEN GOTO 10
160 IF ANS$ < > "N" THEN GOTO 130
170 END
```

2. MAC 8, Program 6 "Number Stumper".

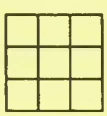
3. "Problem Solving Challenge for Mathematics" p. 41 Number 25; p. 43 Number 3; p. 49 Number 3, 9.

OBJECTIVE A. Whole Numbers	Hm 8	JM 8
4. Understands and uses the terms exponent, base, power, squared and cubed and the nth power of a number.	15, 30, 31, 34, 187	16, 17, 331

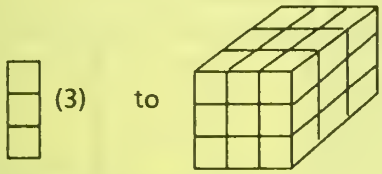
CLARIFICATION OR EXAMPLE

Write the value of a power with a whole number base and exponent (see Grade 7 notes on Whole Numbers Objective #4).

1. Use blocks as manipulatives to develop concepts of making squares.

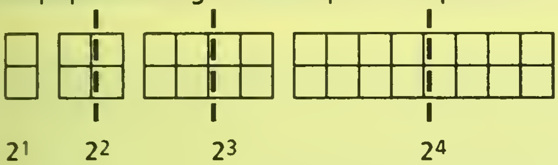
 (3) to  (9)

or making cubes



(See Holtmath 7 Teacher's Edition, p. 26.)

2. Use paper folding to develop the n^{th} power



OR

If $2^2 =$  and $2^n =$  find the value of n.

ELECTIVE SUGGESTIONS

(E) Continue development to encompass square roots using blocks. Have students construct squares; use area and dimensions to develop an understanding of a square root.

(E) Develop law of multiplication with exponents.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Using grid paper on the overhead ask students to use their calculators to guess and check and find *n*.

   e.g., if $2n = 1064$
   $n = ?$

2. Use calculators in a 'timed' activity to reinforce concept.

3. MAC 8, Program 2A "Power Patterns".

OBJECTIVE A. Whole Numbers	Hm 8	JM 8
5. Demonstrates the need for scientific notation.	56	56, 57

CLARIFICATION OR EXAMPLE

See: Holtmath 8 (Teacher's Edition) pp. 56-57.
Journeys in Math 8 (TRM) p. 71.

ELECTIVE SUGGESTIONS

(R) Round off numbers to approximate answers. Multiply whole numbers and count zeros to avoid writing all of the zeros.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Use large numbers and calculators to emphasize the need for scientific notation.

   e.g., $87000 \times 5670 = n$

2. Discuss calculator output such as:

   - 49 329 000 E where multiplication result is too large for calculator to handle
   - 4.9329 8 represents scientific notation as 4.9329×10^8.

3. Use computer to type the following command –
   PRINT 670 924 000 – discuss its output of 6.70 924E + 08.

OBJECTIVE A. Whole Numbers	Hm 8	JM 8
6. Writes numbers in scientific notation, and scientific notation numbers in standard form (limit: positive exponents).	56, 57	56, 57

CLARIFICATION OR EXAMPLE

See: Holtmath 8 Teacher's Edition, pp. 56-57.
Journeys in Math 8 TRM, p. 71.

ELECTIVE SUGGESTIONS

Students work in pairs to make and use flashcards to convert numbers from scientific notation to standard form and vice versa.

e.g.,

FRONT	BACK
6×10^8	600 000 000

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE B. Integers	Hm 8	JM 8
1. Maintains previously developed skills with integers (need for integers, concept of integers, ordering of integers, demonstrates addition of integers with manipulatives).	253, 256-259, 261	273-277

CLARIFICATION OR EXAMPLE

See Grade 7 notes.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE B. Integers	Hm 8	JM 8
2. States the additive inverse of any integer.	266	

CLARIFICATION OR EXAMPLE

Use the terms "additive inverse" and "opposite" interchangeably. Discuss temperature, bank account and above/below sea level to develop the concept of additive inverse/opposite.

Students should understand that the additive inverse is a tool for computation.

(Number + Additive Inverse = 0)

("Concrete Development" Journeys in Math 7 TRM, p. 302.)

ELECTIVE SUGGESTIONS

(R) Use a strip of paper to develop a number line.

e.g.,

−4	−3	−2	−1	0	1	2	3	4

↑

Fold on zero, and find the additive inverse.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE B. Integers	Hm 8	JM 8
3. **Uses concrete manipulatives to demonstrate the subtraction, multiplication and division of integers.**	263	

CLARIFICATION OR EXAMPLE

Concrete development (subtraction). (Journeys in Math 7 TRM, pp. 304-305.)

Extend the idea of changes and coloured chips to multiplication and division.

Multiplication and Division

Emphasize the meaning of the negative sign (opposite). Give each student a red (−) and a black (+) chip. Ask the students to replace the black chip with the opposite colour (one negative sign). What would happen if they were asked to replace the chip with the opposite colour two times (two negative signs)? Three times? Four times? The notion that pairs of opposites (negative signs) cancel each other should immerge.

Begin by using the black chips (+) to demonstrate multiplication as repeated addition and division as repeated subtraction. The colour of the chips to be used in the demonstration will change as integral values are introduced and will be determined by the number of negatives (opposites) in the question (e.g., $2 \times {}^{-}3$ means one opposite, or that red chips will be used in the demonstration).

Note: Students must write a number sentence to describe each operation or activity.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE B. Integers

4. Performs the operations of addition, subtraction, multiplication and division with integers using paper-and-pencil algorithms, estimation, mental computation and a calculator.

Hm 8	JM 8
262-266 268-271 280	280-283 285-289

CLARIFICATION OR EXAMPLE

Concrete activities should be extended to determine strategies for computing without manipulatives. As students write number sentences describing concrete operations, they should be looking for patterns that would enable them to perform the operations without the manipulatives.

An equal emphasis should be placed on mental computation, pencil-and-paper computation, estimation and using the calculator. Timed activities and games may be used to encourage mental facility.

ELECTIVE SUGGESTIONS

(E) Journeys in Math 7 Teaching Aids; p. 81 Game #21, p. 82 Games #23 and 24.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Students should be taught how to compute with integers on a calculator. Because calculators may vary in their operating systems, some time should be taken to investigate the change sign key and to verify the operating system of the calculator. Students should be encouraged to verbalize how their own calculators work when operating with integers.

   Additional: See Journeys in Math 7 TRM, p. 309.
   Holtmath 7, p. 274-275.

2. MAC 8, Program 3 "Number Line Hop".

3. MAC 7, Program 14 "Operation Integer".

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
1. Maintains previously developed skills with decimal numbers (place value, operations, ordering, rounding, order of operations).	37, 40-42, 44-52, 54-55, 122	35-45 30-55

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
2. Maintains previously developed skills with fractional numbers (concept of a fraction, equivalent fraction, basic fraction, mixed numbers, improper fraction, ordering fractions, concrete operations with fractions, order of operations).	101, 104-109	133, 136-139, 142, 143

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
3. Writes the multiplicative inverse (reciprocal) of a fraction, whole number or integer.	119	150, 177

CLARIFICATION OR EXAMPLE

Begin by developing the concept of reciprocals for proper fractions only. Students should understand how the reciprocal/multiplicative inverse acts as a tool (number × reciprocal = 1).

Get students to realize that dividing by 2 is the same as taking one half of an item. Progress to show $\div 3 = \times \frac{1}{3}$, $\div 7 = \times \frac{1}{7}$. Discuss the relationships between 2 with $\frac{1}{2}$, 3 with $\frac{1}{3}$, and 7 with $\frac{1}{7}$.

Extend to what happens to $\frac{2}{3}$.

Progress to show:

$\div 3 = \times \frac{1}{3}$

$\div 7 = \times \frac{1}{7}$

Discuss the relationships between 2 and $\frac{1}{2}$.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
4. Performs the operations of addition, subtraction, multiplication and division with fractions (limit: positive rationals).	112-117, 120, 121, 126	140, 141, 146-151

CLARIFICATION OR EXAMPLE

Review the four operations with fractions at a concrete level. Students should be required to write number sentences to describe the concrete operations while looking for patterns that would enable them to perform the operations without using manipulatives. The fractions used to perform initial paper-and-pencil or mental operations should be relatively simple and should easily be related to a concrete manipulative (especially if difficulties occur). Estimation of answers to computations with fractions should be encouraged continuously.

ELECTIVE SUGGESTIONS

(R) Students should continue to use manipulatives or pictorial presentation until they are able to perform operations formally.

(R) Encourage the students to make and use fraction slide rules.

(E) Develop games like fraction dominos.

(E) Extend strategies developed to include mixed numbers.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 8, Program 7 "Operating on Fractions".
2. "Problem Solving Challenge for Mathematics" p. 43 Number 5.

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
5. Demonstrates the need for rational numbers (e.g., subdivides a unit of measure).	229, 278	294

CLARIFICATION OR EXAMPLE

Discuss examples of uses of rational numbers:

e.g., Measurement
Banking – deposits, withdrawals, overdrafts
Business – profits, losses
Emphasize those examples (e.g., money) that use parts of a whole (rational numbers) and discuss the need for both positive and negative fractional values (rationals).

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
6. Recognizes rational numbers as all numbers that can be written in the form: $\frac{a}{b}$ where $b \neq 0$.	278	294, 295

CLARIFICATION OR EXAMPLE

It is important that students understand that ***numbers have many different but equivalent forms***. Have the students construct a fraction tape (a strip of paper folded in halves, quarters, eighths; thirds, sixths, etc.) and mark the folds as one would mark a number line (zero in the middle, with rational numbers describing the folds on each side). Then ask the students to write the decimal and, if possible, integral equivalents to the rational numbers.

Discuss the fact that any number (regardless of its form) that can be written as a fraction or $\frac{a}{b}$ where $b \neq 0$, is by definition a rational number. Use a calculator to explore the various patterns formed by fractions and their equivalent decimal forms
(e.g., $\frac{1}{7}$, $\frac{2}{7}$, etc., $\frac{1}{9}$, $\frac{2}{9}$, etc., $\frac{1}{11}$, $\frac{2}{11}$, etc.).

ELECTIVE SUGGESTIONS

(E) Are there numbers that do not have fractional equivalents? What are these numbers (irrational numbers)? How are they generated? (See Journeys in Math 8 TRM, p. 157.)

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers	Hm 8	JM 8
7. Compares and orders rational numbers using <, > or =.	41, 110, 111, 278, 279	294, 295

CLARIFICATION OR EXAMPLE

Use the fraction tape (Objective #6) to compare and order rational numbers. It should be made clear to students that on a number line, numbers on the right are larger than numbers on the left (e.g., $1 > {}^{-}3$).

ELECTIVE SUGGESTIONS

(R) Use calculator to make an equivalent form to explore differences.

(E) See Journeys in Math 7 TRM, pp. 174 – 175.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE C. Rational Numbers

8. Uses a number line to demonstrate the relationship between whole numbers, integers, fractions and rationals.

Hm 8	JM 8
260, 278	

CLARIFICATION OR EXAMPLE

Extend the fraction tape. Discuss the relationship between whole numbers, integers, fractions and rationals.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

RATIO

and

PROPORTION

8

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 8	JM 8
	154, 155	174, 175

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 30 Problem 8.15; p. 40 Number 8; p. 43 Number 1.

OBJECTIVE 2. Maintains previously developed skills (understands and constructs ratios, equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another).	Hm 8	JM 8
	131, 134-139, 140, 141, 148-153	167-173, 197-207, 211-213

CLARIFICATION OR EXAMPLE

1. Use time and/or verbal activities to reinforce concepts.

   e.g., a) "Give two equivalent ratios for..."
   b) "Express this ratio in three ways..."
   c) Find the unknown

   $\frac{3}{4} = \frac{x}{20}$ using two methods.

   d) Advertisements that illustrate percentage discounts.
   e) Estimate percentages of test scores and check using a calculator.

A practical example is mixing photography chemicals,

e.g., a) 1 part of water : 9 parts of developer
b) 2 parts of w : 18 parts of d
c) 3 parts of w : ? parts of d.

Students should be encouraged to suggest other practical examples.

LECTIVE SUGGESTIONS

ote: ***It is important that students understand the difference between a fraction and a ratio.*** (Fractions e parts of a whole; ratios are comparisons of two numbers.)
g., In a group of 2 boys and 3 girls, the ratio of boys to girls is 2 to 3 or $\frac{2}{3}$, but the fraction of boys in the group is $\frac{2}{5}$.

) For concrete development of these skills see detailed explanation at the Grade 7 level.

) Students can investigate trends in the stock market.

If a written report is required, the criteria for evaluation must be clearly stated.

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Students could use software from MAC Courseware 8, Program 9A "Ratio Rendezvous" and Program 9B "Pulling Percents" (this will help students to develop their own method of identifying equivalent ratios).

Calculators: Journeys in Math 8, p. 211

Computer: Holtmath 8, p. 155

OBJECTIVE 3. Gives examples of ratios involving situations where the equivalent percent is greater than 100.	Hm 8	JM 8
	153	198, 199

CLARIFICATION OR EXAMPLE

Discuss what a percent greater than 100 means and its practical occurrences (stock markets, growth in industry, retail sales).

a) Use 10 × 10 grids to demonstrate percents greater than 100. Have students shade the grids to show, for example, 120% (one complete 10 × 10 grid and 20 squares from a second grid). Note that 120% is a number larger than 1 but smaller than 2.

b) Use a chart similar to the one below.

Cost Price	Selling Price	Profit	% Profit (x)
\$20	\$25	\$5	$\frac{5}{20} = \frac{x}{100}$ $\therefore x = 25\%$
\$20	\$30	\$10	$\frac{10}{20}$ $\frac{x}{100}$ $\therefore x = 50\%$
\$20	\$40	\$20	$\frac{20}{20}$ $\frac{x}{100}$ $\therefore x = 100\%$
\$20	\$60	\$40	$\frac{40}{20} = \frac{x}{100}$ $\therefore x = 200\%$

Let students suggest other examples.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Converts mixed numbers to percents and vice versa.	Hm 8	JM 8
	148, 149, 153	200-203

CLARIFICATION OR EXAMPLE

Percents larger than 100 have equivalent mixed number forms. Extend the development from Objective 3 to include many examples of mixed numbers (more than one 10 × 10 grid) and their equivalent percents.

Limit: Whole Number %.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Understanding an operation is a necessary requirement before using a calculator or computer. Before permitting (or instructing) students on using the percent key on the calculator, ask the students to describe (in a paragraph) how the percent key on their own calculator works. In other words, describe the algorithm that the engineer (builder) used to "program" the calculator.

OBJECTIVE 5. Given the percent, determines the missing value in applications such as discounts, increases, decreases, or sales tax.	Hm 8	JM 8
	154-159, 163	211, 214-217, 220-221

CLARIFICATION OR EXAMPLE

Groups of students research discounts, increases, decreases or sales tax, using newspaper advertisements. Classroom discussion can follow, stressing mental computation and estimation. Use the calculator to check; e.g., (best deal - discount).

Ask students to collect daily newspaper articles. Discuss the meanings of:
50% more, 30% discount, 5% sales tax. Follow up with students creating their own advertisements.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. "Problem Solving Challenge for Mathematics" p. 22 problem 7.15.
2. Use the calculator to check real examples of discounts collected from local newspapers. A chart may be used to organize the information.

e.g., a.		e.g., b.	
Item:	Suit	Item:	Stereo
Reg. Price:	$150	Reg. Price:	$280
% Discount:	30%	% Discount:	15%
$ Reduction:	150 × 30% = 45	$ Reduction:	280 × 15% = 42
Sale Price:	150 – 45 = $105	Sale Price:	280 – 42 = $238

Other strategies for determining sale price may be used:

a) Given a % discount means having to pay the complement of the discount (up to 100%)

A discount of 30% means having to pay 70% of the original price.
Sale Price: 150 x 70% = $105

A discount of 15% means having to pay 85% of the original price.
Sale Price: 280 x 85% = $238

b) Some calculators will compute a discount or increase directly. Students may check to see if this feature has been programmed into their calculators by using the following key punch sequence. Emphasize that the program is following a routine noted in the examples above.

150 $\boxed{-}$ 30 $\boxed{\%}$ $\boxed{=}$ 105 OR 280 $\boxed{-}$ 15 $\boxed{\%}$ $\boxed{=}$ 238

Increase example: 3% increase in a price of $86

86 $\boxed{+}$ 3 $\boxed{\%}$ $\boxed{=}$ 88.58

Note: $\boxed{\%}$ Represents the percent key on calculators.

OBJECTIVE 6. Understands and writes rates as the comparison of two numbers with different units (e.g., 15 km/2h or 3 items/$1).	Hm 8	JM 8
	140	178-181

CLARIFICATION OR EXAMPLE

See: Holtmath 8 Teacher's Edition, pp. 140-141.
Journeys in Math 8 TRM, p. 178.

Discuss the differences and similarities between ratios and rates. (Ratios and rates are both comparisons between two numbers or quantities. However, ratios are comparisons of numbers or quantities with the same units while rates compare quantities with different units).

ELECTIVE SUGGESTIONS

(E) Use statistics in sports to calculate the m.v.p. (most valuable player) for a particular team or sport.

Batting Statistics

	At Bat	Hits
Tom	60	10
Dick	40	8
Harry	50	20

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 7. Writes proportions involving rates.

Hm 8	JM 8
140, 141	178, 179

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

(E) Students compare sizes of a selected product to determine which is the most economical to buy. The intent is for students to find the unit rates (comparison shopping).

(Holtmath 8 Teacher's Edition, pp. 141-145.)

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 8. Finds the missing element in a proportion involving rates.	Hm 8	JM 8
	141, 143, 146, 147	178, 179, 184, 185

CLARIFICATION OR EXAMPLE

See: Holtmath 8 Teacher's Edition, pp. 136-139.
Journeys in Math 8 TRM, p. 178-179.

ELECTIVE SUGGESTIONS

INTEGRATION OR PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 43 Numbers 7, 8.

MEASUREMENT

and

GEOMETRY

8

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 8	JM 8
	66, 83, 90, 97, 168, 169, 192, 205, 220, 336, 337	47, 93, 123, 223, 247, 260, 261

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Maintains previously developed skills (linear, area, volume, capacity and mass units of measure; uses geometric tools to measure line segments and angles and to construct geometric designs; transformational geometry).	Hm 8	JM 8
	63-65, 74-75, 91, 94-96, 167, 170-173, 206, 308-315, 318-321	69-71, 90, 91, 118-121, 233, 236, 237, 375-385, 388-393

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

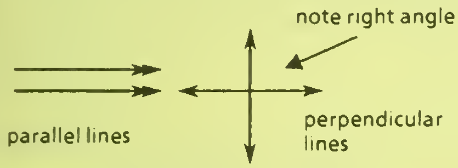
BJECTIVE 3. Understands and uses the terms perpendicular and parallel lines.

Hm 8	JM 8
178-181	234-235

LARIFICATION OR EXAMPLE

See Holtmath 8 Teacher's Edition, pp. 178-181.
Journeys in Math 8 TRM, p. 234.

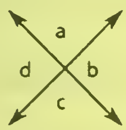
Give pictorial diagrams to show difference between perpendicular and parallel lines.



LECTIVE SUGGESTIONS

:) Explore the relationships between angles of intersecting lines.

e.g.,



a = c, d = b, b + c = 180°.

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

BJECTIVE 4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor.

Hm 8	JM 8
183, 186	91, 115, 245, 260, 261

LARIFICATION OR EXAMPLE

he intent of this objective is to investigate the attributes (characteristics) of polygons which will lead to definition of a polygon (a plane closed figure whose sides are line segments). Further "investigation" f the attributes (e.g., measures of sides, congruence of sides and/or angles, measure of interior angles, umber or length of diagonals) can lead to the classification of the polygons according to the number of ides or as regular which is (objective #5).

Students should be asked to place a number of dots (points) randomly on a paper (plane). Keep the number of points small (2-10). Then ask students to connect the points to form various figures. Discuss the outcome by grouping (classifying) the figures using the following points as a guide:

a) the figure must be closed (have an interior and an exterior)

b) line segments (not curved lines) define the sides of a polygon

c) the minimum number of points required to close a figure is 3.

Further investigation will lead to the classification of the polygons according to the number of sides.

Many techniques or instruments may be used to perform the actual sketches or constructions. Traditional tools such as the compass, protractor or ruler have "built-in" mathematics concepts but their use may be tedious. The computer will allow students (after learning to use a graphics program or a program such as LOGO) to generate many and varying shapes in a short period of time or to "instruct" a computer to construct a polygon. (For more information on using a LOGO program see Journeys in Math 8, pp. 91, 94-95.)

ELECTIVE SUGGESTIONS

The following points may be used as a guide in the investigations and subsequent discussions:

a) congruent sides
b) congruent interior angles
interior
angles

c) both congruent sides and angles are conditions for a regular polygon
d) number of diagonals
e) relationship of the perimeter of a regular polygon to the length of its diagonal.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

The following are references to LOGO procedures which may be generated by students. These procedures will allow students to construct various polygons with a computer (LOGO software is required)

Journey in Math 7, pp. 120-121, 151, 158-158.

Journeys in Math 8, pp. 91, 115.

	Hm 8	JM 8
OBJECTIVE 5. Identifies and classifies polygons according to the number of sides (limit: decagon).	193, 196, 197	245

CLARIFICATION OR EXAMPLE

See Objective #4.

ELECTIVE SUGGESTIONS

(E) Students could investigate to find the names of some polygons with more than 10 sides.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 6.	Hm 8	JM 8
Investigates triangles by examining attributes such as measure of angles, measure of sides and lines of symmetry.	182, 184, 185, 317	242, 243

CLARIFICATION OR EXAMPLE

Investigate triangles by having students construct a number of random triangles. Discuss their similarities and differences and the different attributes (length of sides, measure of angles, number of lines of symmetry) that may be used to classify the triangles (before assigning names to the classifications).

ELECTIVE SUGGESTIONS

(E) See Holtmath 8 Teacher's Edition, pp. 186-187, "Rigid and Non-Rigid Shapes".

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 8, Program 11 "Triangle Tryouts".
2. "Problem Solving Challenge for Mathematics", p. 47 Number 6.

OBJECTIVE 7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals.	Hm 8	JM 8
	194, 195, 317	244, 245

CLARIFICATION OR EXAMPLE

A discovery approach is encouraged. Either provide to students, or have them construct (from black line masters) a "quadrilateral kit". This kit should contain a number of varying shapes that are large enough to measure and manipulate: irregular quadrilaterals, trapezoids, parallelograms, rhombi, rectangles and squares. Discuss the various attributes of the shapes and then have the students group the shapes according to those attributes. Note that some of the quadrilaterals can fit into several categories.

e.g., Quadrilaterals that have: four sides; (at least) one pair of parallel sides; opposite parallel sides; opposite sides are congruent (Can opposite sides of quadrilaterals be parallel without being congruent?); congruent opposite angles; opposite parallel sides and at least one right angle; number of lines of symmetry; number of diagonals; congruent diagonals; diagonals bisect each other; diagonals meet at right angles.

Develop a classification scheme based on the properties of the quadrilaterals, which would relate quadrilaterals, trapezoids, parallelograms, rhombi and squares.

Alternative development:

Journeys in Math 7 TRM, pp. 144-145.

ELECTIVE SUGGESTIONS

(E) Journeys in Math 7 TRM, p. 145.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem Solving: Explore tangram-type puzzles.

OBJECTIVE 8. Adds, subtracts, multiplies and divides using SI units of measure.

Hm 8	JM 8
68	90, 91

CLARIFICATION OR EXAMPLE

Only commonly used units (km, m, cm, and mm) should be used to develop this objective. The operations with units of measure should be developed in the context of real problems (e.g., perimeter and area). To add, subtract, multiply or divide measures, the units must be common (the same) or else the result will not have a standard meaning (e.g., 3 m + 4 cm = ? or 2 cm x 5 mm = ?).

Multiplication (or its inverse-division) of measures is a difficult concept. Students should be taught that cm^2 is an expression of area (a count of square centimetres) rather than meaning cm "times" cm.

This objective may be developed within the context of objectives #9 (perimeter) and #10 (area).

ELECTIVE SUGGESTIONS

(E) Explore the relationship between, for example, $1 m^2$ and cm^2.

(E) Explore the unit of area hectare (ha).

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 9. Understands and uses formulas as indirect measures of the perimeter of polygons (includes regular polygons).

Hm 8	JM 8
69-71	72-75

CLARIFICATION OR EXAMPLE

Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For perimeter this experience may be using a string (and a ruler) to determine the distance around different polygons; then using only a ruler to measure the sides and then determining the sum of the sides. Students should note that the order in which the measures are taken or summed, does not affect the result. Students should verbalize a strategy before attempting to generalize it in the form of a formula. For example, "the perimeter of a rectangle is two lengths added to two widths because . . ." The generalization or the formula should be the last stage of the development and may be used to expedite the process of determining a perimeter. The use of formulas may not be appropriate for all students.

ELECTIVE SUGGESTIONS

(R/E) Journeys in Math 8 TRM, pp. 80-81.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 47, Number 6.

OBJECTIVE 10. Understands and uses formulas as indirect measures of the area of polygons (triangles, all parallelograms and trapezoids).

Hm 8	JM 8
76-82	78-81, 84-87

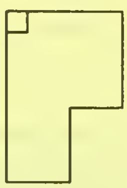
CLARIFICATION OR EXAMPLE

Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For area, this experience is counting the number of squares that can be found within a closed region. Transparency grids (cm × cm) may be used to determine the areas of both regular- and irregular- shaped objects. The measures should then move on to rectangles where it should be discovered that the number of squares in one row corresponds to the length, and the number of rows corresponds to the width. Hence, the product of the length and the width determines area. Students should be encouraged to verbalize the strategy before generalizing it. Areas of other polygons (e.g., triangle, parallelogram) are variations of the rectangle.

(Journeys in Math 8 TRM, pp. 83-87.)

ELECTIVE SUGGESTIONS

(E) Expand basic strategies to finding areas of compound figures.

e.g., 

The area of this can be found by dividing the figure.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 7, Program 5 "Metric Mysteries".
MAC 8, Program 12 "Metric Mysteries".

OBJECTIVE 11. Performs an experiment to determine the value of π and understands π as a ratio of the circumference of a circle divided by its diameter.

(i.e., $\pi = \frac{C}{d}$)

Hm 8	JM 8
72	75, 76

CLARIFICATION OR EXAMPLE

An introductory activity for this objective could be to draw different sizes of a particular polygon and then determine the ratio of the perimeter to the length of a diagonal (have the students take the measurements).

e.g., For a set of squares, $\frac{\text{perimeter}}{\text{diagonal}} = 2.9$

From this activity, the students should find the principle that, if figures have the same shape, the ratios of the corresponding parts remain constant even though the size is different.

Using the above activity, the students might predict that $\frac{C}{d}$ will be a constant (because all circles have the same shape). By measuring various round objects (records, coins, jars, cans) using a string and a ruler (a metre stick, if necessary) and keeping a record of the diameters and circumferences, the concept that π is a ratio of C to d should be discovered.

ELECTIVE SUGGESTIONS

(E) Use the library or other resources to gather information about the historical development of π.

(E) Develop a strategy for finding the diameter of a circle when the centre is not given.

(E) Journeys in Math 8 TRM, p. 82.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem Solving: By comparing measurements, the question of accuracy should arise. When is it important? When is an estimation appropriate?

OBJECTIVE 12. Understands and uses the formula $C = \pi d$ as an indirect measure of the circumference of a circle.	Hm 8	JM 8
	72, 73	76, 77

CLARIFICATION OR EXAMPLE

Using the measurements from Objective #11, compare calculated circumference ($C = \pi d$) to measured circumference. The development of the formula as a strategy should flow directly from the previous topic.

Given the radius, explore how to find the circumference. Two possibilities can be used $C = \pi d$ (double the radius and then utilize the formula), or $C = 2\pi r$.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 44 Problem #4.

OBJECTIVE 13. Uses the formula $A = \pi r^2$ to determine indirectly, the area of a circle given its radius or diameter.	Hm 8	JM 8
	84, 85	88-89

CLARIFICATION OR EXAMPLE

Use a compass and grid paper and have students draw a circle with a specified radius. By counting squares, estimate the area. Compare estimates.

ELECTIVE SUGGESTIONS

(E) Students show development of the formula $A = \pi r^2$. See Journeys in Math 8 p. 88.

(E) Show that the area of a regular polygon ($A = \frac{1}{2} ans$ where *a* is the apothem, *n* is number of sides, and s is the measure of one side) approaches the area of a circle ($A = \pi r^2$) as the number of sides increases.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 24. Problem 8.1.

OBJECTIVE 14. Draws or sketches a right rectangular prism.	Hm 8	JM 8
	218, 219, 335	106, 107

CLARIFICATION OR EXAMPLE

From everyday objects such as a cereal box, or a tissue box, discuss the relationship between sides. From this, students should be able to determine the characteristics of all right rectangular prisms. Prisms are easily drawn by using isometric paper or grid paper.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 15. Understands and uses a formula as an indirect strategy for determining the volume of a right rectangular prism or a cube.	Hm 8	JM 8
	92, 93	114, 115

CLARIFICATION OR EXAMPLE

A formula is an indirect strategy for calculating a measure that can sometimes be directly obtained. Students should have experiences with obtaining the measure directly before determining a strategy for computing the measure. Use centimetre cubes (sugar cubes will do if other manipulatives are unavailable) to determine the volume of rectangular prisms. Note that the number of cubes in one layer corresponds to the area of the base (length times width) and that the number of layers corresponds to the height.

See Holtmath 8 Teacher's Edition, pp. 91-93.
Journeys in Math 8 TRM, p. 102.

ELECTIVE SUGGESTIONS

(E) Journey in Math 8 TRM p.102.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 45 Number 16.

DATA MANAGEMENT

8

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 8	JM 8
	356	319

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

See Grade 7 Data Management Objective #1.

OBJECTIVE 2. Maintains previously developed skills (understands the purpose of statistics; interprets data from tables and graphs; draws graphs).	Hm 8	JM 8
	344, 345, 347, 348-355, 358-359	208-209, 307, 312-317

CLARIFICATION OR EXAMPLE

1. Ask how numbers affect our lives. Use sources such as measurement, experimentation, observation, encyclopedias, newspapers, information services.

2. Recap frequency tables, pictographs, bar graphs, line graphs, and circle graphs as ways of organizing and comparing data.

ELECTIVE SUGGESTIONS

(R) Explain strengths and weaknesses of each graph.

(E) Use histograms, stem-leaf tables, box and whisker plots.

Utilize graphs from newspapers to display on a bulletin board.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Use computer programs that will construct graphs.

OBJECTIVE 3. Understands and uses the terms bias, sample and population.

Hm 8	JM 8
	324, 325

CLARIFICATION OR EXAMPLE

The following definitions are intended for teachers and must be interpreted for students.

A **population** is the total number of individuals, groups or things from which information is collected.

A **sample** is a smaller, representative part of a population.

A **bias** is a misrepresentation or distortion that occurs in collected information as a result of ignoring a factor or a characteristic of a population. A bias occurs when a sample is not representative of a population.

Demonstrate sampling by taking a survey of students (e.g., from one row in a class) who are wearing running shoes, and then compare the sample to the population of the class.

Discuss the bias by asking if this sample is likely to be representative of the school population (including the teachers) and of the town or municipality.

Discuss how to obtain a useful sample in each case.

The same type of activity could be done to compare the number of people who wear glasses to those who do not.

SELECTIVE SUGGESTIONS

(i) Direct three or four pairs of students to complete a poll based on ten responses to the question, "Do you like orange juice or apple juice?" Ask students why results may vary.

(ii) Get students to conduct polls, interpret the results, and discuss problems that could arise.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem Solving: How are the fish in a lake counted?

OBJECTIVE 4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey).	Hm 8	JM 8
		324

CLARIFICATION OR EXAMPLE

A survey is a random poll of an identified group for the purposes of collecting data or acquiring information about some aspect of the group or area. A survey may be biased if the sample was not random (i.e., if the sample was not representative of the population) or if the sample included a population outside the identified group. For example, a survey to determine the most popular television program among junior high students would be biased if only one class of Grade 7 students were polled (only one age group was represented in the sample) or if some elementary or high school students were included in the sample.

A census is a count or poll of an entire population and provides accurate information. A census is not always practical because of the size of the identified group and the time and costs associated with the gathering of information.

Discuss the merits of a survey and a census and the situations or conditions that would be required to make a census or survey appropriate. When is accuracy more important (population of a school to determine per-pupil grants) and when is knowing the information quickly more important (deciding how many hot dogs and how many hamburgers should be purchased for a school function)?

ELECTIVE SUGGESTIONS

Discuss how a survey may be conducted.

a) How does one conduct a survey: personal interviews? questionnaires? telephone calls? interviews?
b) Discuss whether answers to surveys should be restricted or unlimited.
c) Discuss problems associated with census.

A discussion on how to ask questions could lead to realization that questions can influence the outcome of a survey?

e.g., Do you feel the current admission to movies is too high? OR What is your feeling on the proposed hike in movie admission?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. A data base or spreadsheet like the Appleworks program can be used to sort information and/or 'pull out' the required information.

2. Mindscape Computer Program.

BJECTIVE 5. Recognizes the use and misuse of statistics in society (news reporting, census, polls, etc.).	Hm 8	JM 8
	358-359	

LARIFICATION OR EXAMPLE

atistics are used to predict outcomes but one questions their reliability as one finds incomplete, isleading, and misrepresented data. Find examples of how one uses statistics to one's advantage.

g., Political survey polls which predict party support are often based on a limited sample which could create varying outcomes.

LECTIVE SUGGESTIONS

) Predicting the standing of a sports team (baseball, hockey, local team) based on its record in the last ten (x) games may be misleading. A team with a good record in the last ten (x) games may still end up last in the standings over the whole season.

) Watch a newscast on TV to determine where statistics are used, how, why, and discuss their accuracy and timelines, etc.

ITEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

ALGEBRA

8

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 8	JM 8
	210	368, 369

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Maintains previously developed skills (variable, evaluation of expressions, concept of equality, plots on a coordinate plane).	Hm 8	JM 8
	225-227, 229, 296, 297	278, 279, 310, 311, 341, 345

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 3. Identifies and combines like terms.	Hm 8	JM 8
	238	346, 347

CLARIFICATION OR EXAMPLE

Possible development on concrete level.

3 apples and 5 apples is 8 apples
3 apples and 5 oranges = ?
Alternate manipulative strategy:
Use algebra tiles or base 10 blocks.

ELECTIVE SUGGESTIONS

(E) Use formal approach to explain answers to $x + 3x$, $7n - 3n$, $8y - 9y$. Expand into fractional variables.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Uses formal procedures to solve equations of the form $x + a = b$, $ax = b$, $ax + b = c$, $ax + bx = c$, and $\frac{x}{a} = \frac{b}{c}$ (limit: positive rational numbers and integers).	Hm 8	JM 8
	232-237, 242-245, 248	348-351, 353-359

CLARIFICATION OR EXAMPLE

Review the concept of equality. (See Objective #6, Algebra Grade 7.)

Formal algebraic techniques may be introduced by using the concept of balance and so a balance scale lends itself here. For example, a nickel and 2 one-gram masses balance with 7 one-gram masses. The mass of the nickel (represented by x) can be found by removing 2 one-gram masses from each side (the opposite operation).

ELECTIVE SUGGESTIONS

(E) MAC 8, Program 8 "Eckses and Ohzs".

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 5. Verifies solutions to the equations.	Hm 8	JM 8
	230-231	348, 349

CLARIFICATION OR EXAMPLE

Review the concept of evaluating expressions.

Verifying a solution to an equation is simply a matter of using the solution independently to evaluate the expression on each side of the equation to determine if in fact the LHS (left-hand side) is equal to the RHS (right-hand side).

ELECTIVE SUGGESTIONS

(E) Estimate solutions by guess and check strategy.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 6.	Uses substitution and equation-solving techniques to find a missing element of a formula: e.g., If p = 2 and q = 0.5 find c in $p = \frac{c}{q}$	Hm 8	JM 8
		168, 169	281, 344, 345

CLARIFICATION OR EXAMPLE

The intent of this objective is to de-mystify formulas. The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., d = rt is used to find distance, rate or time). The result is that little transfer occurs in knowing how to use other formulas in mathematics and other disciplines.

A much more holistic view of formulas must be taken. The notion that relationships among real-world values (e.g., time, speed, distance) can be described mathematically should be emphasized. The outcome of this objective should be that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used), and will know how to substitute and solve for missing elements.

ELECTIVE SUGGESTIONS

(E) Have students write their own nonsensical formulas and a funny story about what the variables represent. Exchange the formulae and stories within the class.

For example: $E = 3 + 7a$ where E represents the number of elephants and *a* is the number of people with blue eyes.

(E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc.

(R) Use real life situations where 3 apples and 4 oranges cost a certain amount of money. Give variations for 2 of the elements so the third element needs to be solved.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 7. Generates a set of ordered pairs in a linear relation.	Hm 8	JM 8
	290-295	310, 311

CLARIFICATION OR EXAMPLE

THE GRAPHING COMPONENT OF THE ALGEBRA STRAND SHOULD BE DEVELOPED (AT ALL THREE GRADES LEVELS) WITH THE FOLLOWING CONTEXT IN MIND:

Numbers relate to each other. The emphasis in this program is placed on the understanding of algebra as a generalization of the relationships and patterns of pairs of numbers. Generally speaking, early elementary school mathematics deals with single numbers and their recognition and operations. The focus of secondary school mathematics shifts to 'pairs of numbers' and the relationships that exist between those pairs. Coordinate geometry (graphing), the use of formulas and eventually the study of trigonometric functions, polynomial functions, logarithmic functions, etc., become central to the secondary program.

Students should learn that the relationships occur naturally and are pervasive. They should feel comfortable in describing them. By the end of Grade 9, students should be able to identify the relations that are functions.

Relations are defined as the one-to-one correspondence that exist between two elements or two sets of data. Associating a name with an object, completing a 'times' or 'add' table, comparing the height to weight of people or comparing the area of a circle to its radius, are all relations.

Functions are those relations where the value of the first element determines the value of the second. Given the first element, only one possible pair of numbers can satisfy the condition of the function. The value of the second element "depends" on the value of the first element.

Example 1: When given the radius of a circle, only one value can describe the area. Area is a function of the radius.

Example 2: While the height of a person is related to weight, the relationship is **not** a functional one. A unique pair of numbers cannot describe the relationship (i.e., a person with a given height will not necessarily be a specific weight).

Graphs are pictoral representations of relations. Two types of skills are required for graphing. The first is the **technical skill** of choosing a scale, plotting points and drawing the lines (or curves). The second is the **interpretive skill** that contains elements such as increase, decrease, maximum, minimum, rate of change and slope. Because graphs contain a great deal of information in a relatively small amount of space, the interpretive skills are generally more difficult to deal with and are often neglected.

Grade 7 – technical skills (plotting points on a coordinate plane, choosing a scale, and drawing the graph).

Grade 8 – technical/interpretive skills (constructing a table of values that identify a function and graphing the relation).

Grade 8 – (cont'd)

Review the concepts of a mathematical relation. Note the similarities and differences of several different relationships (e.g., height and age of students; skill (in sport, playing an instrument, etc.) and amount of practise; the total cost of a multiple purchase of a single item (e.g., pairs of socks); the cost of a taxi ride and distance). The concept of function should begin to emerge: i.e., with some of the relationships a rule can be used to describe the relationship when one of the values is given; and there can be only one possible value for the second element when the first is given. In other relationships, while the value of one element depends on the other, there is no hard and fast rule to describe the relationship. (e.g., skill depends on practise).

Use several practical examples of (linear) relations. Have students determine the function rule that determines the relationship (e.g., perimeter of a circle depends on the radius; in this case the function is a formula; multiple purchase of a single item) and then generate the ordered pair that describes the relationship for any given value.

Objective 8 (constructing a table of values and a graph) is the logical extension of this objective.

Grade 9 – interpretive skills (given a graph or table of values, identify the function). (Limited to linear relations.)

ELECTIVE SUGGESTIONS

(E) Given the ordered pairs, determine the relation.

(R) Give values for x as 0, 1, and 2 or numbers that are conducive to easy solution.

(E) Develop a computer program to generate ordered pairs.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

(R) Use a computer to give the pairs.

```
  5 REM ordered pairs
 10 HOME
 20 PRINT "This program will generate ordered pairs for relation rules in the form ax + b."
 30 PRINT:PRINT "You will be asked to give values of a and b, and then the computer will print
    values for ax + b by using a range of –10 to + 10 for x."
 40 INPUT    "What value of a?";A
 50 INPUT "What value of b?";B
 60 HOME
 70 PRINT "X", A; "X + ";B
 80 PRINT "_ _ _ _ _ _ _ _ _ _ _ _ "
 90 FOR X = –10 TO 10
100 PRINT X, A*X + B
110 NEXT
120 END
```

OBJECTIVE 8. Given a linear relation, constructs a table of values and a graph for that relation.	Hm 8	JM 8
	298-301 p. 304-305 Gr. 9.	310, 311

CLARIFICATION OR EXAMPLE

See "Clarification or Example" Objective 7.

ELECTIVE SUGGESTIONS

(E) Given a linear relation of the form $y = ax + b$, explore how changing one aspect (a or b) of the relation changes the graph.

(E) Have students write relations and exchange these in the class for evaluating and graphing.

(R) Give a table of values for students to construct a graph on grid paper.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 8, Program 14 "Activities Courseware Solutions: One Equation".

GRADE 9

PROBLEM SOLVING

9

OBJECTIVE 1. Demonstrates an understanding of a problem-solving situation.	Hm 9	JM 9	PSCM
	9, 30-31	30-32	p.8

CLARIFICATION OR EXAMPLE

Refer to introductory comments under Objective #1 in Grade 7 or 8.

ELECTIVE SUGGESTIONS

Refer to introductory comments under Objective #1 in Grade 7 or 8.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Refer to introductory comments under Objective #1 in Grade 7 or 8.

OBJECTIVE 2. Demonstrates a willingness to find a solution to a problem.	Hm 9	JM 9
		32, 58, 88

CLARIFICATION OR EXAMPLE

Refer to introductory comments under Objective #2 in Grade 7 or 8.

ELECTIVE SUGGESTIONS

Refer to introductory comments under Objective #2 in Grade 7 or 8.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Refer to introductory comments under Objective #2 in Grade 7 or 8.

OBJECTIVE 3. Uses a variety of strategies to solve problems. Previously developed strategies are used.	Hm 9	JM 9	PSCM
	22-23, 58-59, 90-91, 134-135, 180-181, 242-243, 290-291, 318-319, 360, 391	24-25, 56-58, 86-87 118-119, 152-153, 222-224, 260-263, 298-300	p.43(2) p.46(4, 5) p.49(7)

CLARIFICATION OR EXAMPLE

As students encounter more complex problems, the skills required to solve them become more intellectually complex. Consequently, students must use higher-level thinking skills such as logic or reasoning.

To introduce the strategies of problem solving, an approach such as the following may be used: choose three non-related but similar problems that can be solved focussing on a strategy such as using logic or reason. (Consider that any problem usually requires the application of more than one strategy.)

For students to become independent problem solvers the first problem could be a teacher demonstration, the second could be a student trial with teacher guidance, and the third could be student practise.

1. Teacher Demonstration

   Margie is a blonde, Rose Mary a redhead, and Shirley is a brunette. They are married to Alex, Frank, and John but

   a) Shirley does not like John.

   b) Rose Mary is married to John's brother.

   c) Alex is married to Rose Mary's sister.

   Who is married to whom? Assume that married people like each other! ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 40, #17.)

2. Teacher Guidance

   There are eight baseballs, all exactly alike in size and appearance, but one is heavier than any of the other seven which are all the same weight. With a balance scale, how can the heaviest baseball be positively determined with only two weighings? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 35, #9.9.)

3. Student Practice

(The natural progression leads to students using the same strategy to solve problems.)

a) A ferryboat, when filled, can carry 6 Pintos and 7 Toyotas or 8 Pintos and 4 Toyotas. If the ferryboat carries Toyotas only, then what is the maximum number that it can carry? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 42, #39.)

b) Bill, John, Joe, and Henry have to catch the six o'clock bus.

i) Bill's watch is 10 minutes fast, but he thinks it is 5 minutes slow.

ii) John's watch is 10 minutes slow, but he thinks it is 10 minutes fast.

iii) Joe's watch is 5 minutes slow, but he thinks it is 10 minutes fast.

iv) Henry's watch is 5 minutes fast, but he is under the impression it is 10 minutes slow.

If each leaves to catch the bus he will just make it, if his time is what he thinks it is. Who misses the bus? ("Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 44, #5.)

Evaluation

To evaluation of problem solving requires more than grading the solution to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.

a) willingness to attempt problems

b) use of systematic approach

c) selection of appropriate strategies strategies

d) logical justification of strategies and solutions

e) growth in confidence in problem-solving ability

f) transfer of problem-solving skills to situations other than mathematics.

Evaluation techniques and instruments for problem solving are found in "Problem Solving Challenge for Mathematics"(Alberta Education), 1985, pp. 7, 8, 52-56.

ELECTIVE SUGGESTIONS

Problem-solving skills are essential for all students; being perplexed when first encountering a problem, is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success. It is very important to recognize individual student differences in learning; therefore the growth expectations should also vary. Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.

A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider other possibilities such as:

- other strategies and solutions
- "what if?" (change an element of the problem)
- generalization of rules to other situations.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used.

2. Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

3. The use of relevant and realistic problems (from local sources such as newspapers and magazines) is encouraged because this will increase the interest of the students.

   Students may contribute their own ideas of problems; in addition to collecting data they may make up questions related to the information. An exchange of problems and questions may be encouraged.

4. Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies.

   Examples can be obtained from such sources as: Mathematics Activity Software (MAC) by Houghton Mifflin, Minnesota Educational Computing Consortium (available from ACCESS) and Sunburst Communications Software.

OBJECTIVE 3: Uses a variety of strategies to solve problems. Previously developed stratgegies are used.	Hm 9	JM 9	PSCM
The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:			
a) Understanding the problem			
• considers alternative interpretations			
• makes assumptions		32	
b) Developing a plan (choosing a strategy)			
• formulates an equation	121, 134-135	176-177	25(8.5) 26(8.6) 32(9.3) 36(9.12) 43(4) 45(10) 46(3) 49(12)
• uses logic or reason	58-59		24(8.3) 25(8.4) 32(9.4) 35(9.9) 39(5) 40(7, 9) 44(5) 46(8) 47(8) 48(18)
• constructs flow charts			
• develops a symbol or code system			
• recognizes limits and eliminates possibilities			

Hm9 (Holtmath 9)
JM 9 (Journeys in Math 9)
PSCM (Problem Solving in Mathematics

	Hm 9	JM 9	PSCM
c) Carrying out the plan			
• applies selected strategies	169	32, 58, 88, 120, 154, 176-177	
• presents ideas clearly			
• documents the process			
• works with care			
• works in a group situation			
d) Looking back			
• generalizes solutions			
• creates and writes routine and non-routine problems	90-91 180-181		

NUMBER SYSTEMS

and

OPERATIONS

9

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 9	JM 9
	58-59	32, 56-58, 67, 73, 120

CLARIFICATION OR EXAMPLE

The intent of this objective, placed at the beginning of each strand, is to reinforce the fact that growth in students' ability to solve problems is a major goal of the program. Problem solving should not be viewed as an isolated activity but, rather, as a group of related activities, skills and attitudes that enhance students' capability to work in new or unfamiliar situation. A student's perplexity about a newly introduced concept or his/her inability to answer a question should be treated as a normal state, in a problem–solving environment. The emphasis must be placed, not on finding a singular solution or strategy, but on the development of several strategies for understanding, or for working towards a solution. The development of the knowledge, skills and attitudes associated with working in new or unfamiliar situations should become part of the teaching philosophy.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	Hm 9	JM 8
	2-3, 5, 15, 17, 45, 63, 83, 91, 96, 127, 223, 277, 287, 307, 381	4-7, 14, 15, 77, 135

CLARIFICATION OR EXAMPLE

An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping or borrowing, and place value. Long and tiresome paper-and-pencil drill is discouraged.

Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation and knowing when a computed answer is possible, are among skills to be emphasized.

Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve.

Calculators should be used to develop understanding, to investigate patterns, and to perform tedious computations that do not enhance understanding.

ELECTIVE SUGGESTIONS

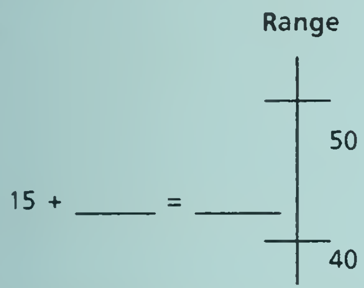
INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

A good calculator estimation activity is the "Range Game". Estimation and Mental Computation. Reston, VA: National Council of Teachers of Mathematics, 1986, pp. 182-185.

"The Range Game"

"The Range Game" builds estimation skills. The only tools needed are a calculator and paper and pencil for record keeping. The object is to find numbers within a given range that will satisfy a given equation. Introduce the game by displaying a range and a partial equation, as shown below.

Example:



Ask questions such as the following:

a) "Find a number that when added to 15 gives a sum in the range shown."

b) "What is the largest number that works?" "The smallest?"

c) "Are there any other numbers?"

d) "Let's find all the numbers that will work."

List responses on the board and discuss the findings. Ask, "How many numbers work?"

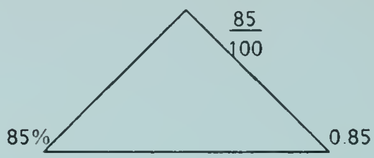
OBJECTIVE 3. Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations).

Hm 9	JM 9
1, 4-8, 29, 32-41, 43-45, 57, 60-63, 68, 153, 168, 175	1, 12-14, 16, 17, 20-23, 35, 36-44, 46-52, 63, 93, 125

CLARIFICATION OR EXAMPLE

Use time drills with basic number facts, in particular multiplication and addition. Encourage estimation, mental computation and the use of a calculator.

e.g., An activity that demonstrates a relationship among number systems is TRIPLE CHECK



Usually by the first of March, my sixth-grade pupils have demonstrated that given a rational number in any of the three forms, fraction, decimal, or percent, they can quickly give the two equivalent names for commonly used fractions and can find equivalents for other fractions, decimals, or percents.

To maintain and increase that knowledge, we use Triple Check at various intervals throughout the rest of the school year. On the last day of each month, pupils give their daily attendance, reading and mathematics grades, and spelling scores in "Triple Check". Usually we begin with a fraction, for example, for 17 days of attendance out of a possible 20 days, we would write $\frac{17}{20} = \frac{85}{100} = 85\% = 0.85$.

However, if you are working with decimals or percents, name them first.

Triple Check encourages pupils to check their answers in a reasonable and an exact manner. It reinforces the idea that fractions, decimals, and percents communicate the same information about rational numbers.

(From the file of Celestine Wyatt, 10629 S. Emerald, Chicago, IL 60628. Arithmetic Teacher.)*

ELECTIVE SUGGESTIONS

(R) Rather than doing the actual division, ask students to write the number of digits in the whole number part of the quotient.

e.g., $0.98\overline{)10.03}$ 2 digits

(R) Ask students to place decimal points in four different ways to make four true number sentences in each set.

e.g., 124 × 62 = 7688 124 × 62 = 7688
124 × 62 = 7688 124 × 62 = 7688

(Idea from Arithmetic Teacher, Vol. 34, #7, March 1987.)*

(E) Investigate the stock market, 'Why are fractions used rather than decimals?'

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Suggest calculator problems such as those from NCTM "Activities for Junior High School and Middle School Mathematics", p. 202. Play a game called "Erase". Have your friend enter any six-digit number into the calculator. Now see what is the fewest number of moves it takes you to get to a display of zero. For each move, you may add, subtract, multiply, or divide by any two-digit, non-zero number.

Trade places and have your friend try to "erase" a six-digit number you enter into the calculator.

What did you notice about numbers that make them easy or difficult to erase?

(From NCTM "Activities for Junior High and Middle School Mathematics", p. 202.)

OBJECTIVE 4. Performs the operations of addition, subtraction, multiplication and division with rational numbers.

Hm 9	JM 9
64-67, 72-73	68-72

CLARIFICATION OR EXAMPLE

Review the operations with positive rational numbers (see Grade 8 Objective #4) and the operations with integers (see Grade 8 Objective #3).

Concrete activity:

$-2\frac{3}{4} + 1\frac{1}{4}$

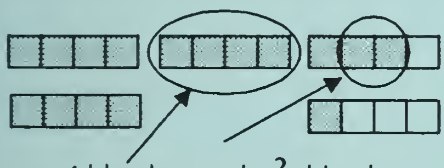
Use manipulatives consisting of rectangular shapes divided into quarters.

Blue 
(negative rational)

Red 
(positive rational)

To find the answer to the above problem balance as many complete boxes and quarters of boxes as possible.

e.g., 1 red box balances 1 blue box.
$\frac{1}{4}$ red box balances $\frac{1}{4}$ blue box.

Blue
Red


This leaves 1 blue box and $\frac{2}{4}$ blue box or ($-1\frac{2}{4} = -1\frac{1}{2}$)

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

MAC 8, Program 7 "Operating on Fractions".

OBJECTIVE 5. Applies the rules for order of operations to evaluate expressions involving rational numbers in any of their forms.

Hm 9	JM 9
70-71	74-75

CLARIFICATION OR EXAMPLE

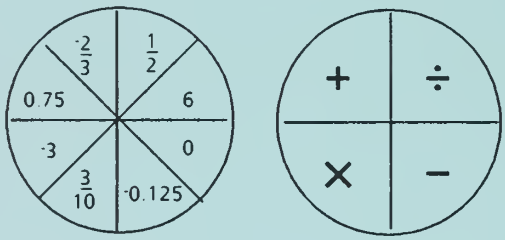
Review the rules for order of operations with whole numbers, fractions and integers. Extend the concepts to rational numbers.

ELECTIVE SUGGESTIONS

(R) Working in groups of two, develop spin game. Teacher instructs how many spins of each. A problem involving order of operations emerges.

Have students estimate and then calculate their answers. A discussion will follow focusing on order of operations rules.

e.g.,



Encourage the use of calculators.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Explore the operating system of a calculator by entering an operation in sequence. Ask the students to explain the operating system of their own calculator (e.g., Is the order of operation "built in"?).

OBJECTIVE 6. Converts rational numbers from $\frac{a}{b}$ form to decimal form (limit: $b < 10$ or b is a power of 10).

Hm 9	JM 9
74-75	78-79

CLARIFICATION OR EXAMPLE

Students have converted rational numbers in the form $\frac{a}{b}$ to decimal form by converting to denominators of 10, 100, or 1000 and by using a calculator.

Students should now be introduced to the paper-and-pencil algorithm of dividing the numerator by the denominator.

Stress should be placed on the meaning of the fraction bar (3 out of 4, three quarters, or 3 divided by 4).

Emphasis should be placed on the following:

1. meaning of symbols ÷ and the – (bar) in $\frac{a}{b}$.

2. numbers can have different forms (decimal, fraction percent).

ELECTIVE SUGGESTIONS

(E) Explore the resulting decimal forms of different rationals: terminating, non-terminating, repeating or non-repeating. In particular an interesting activity is to contrast fractions with denominators of 11 to fractions with denominators of 9.

(E) Post this activity on the bulletin board but let students determine the answers.

D	100th digit of $\frac{1}{D}$	D	100th digit of $\frac{1}{D}$
2	0	11	9
3	3	12	3
4	0	13	9
5	0	14	4
6	6	15	6
7	8	16	0
8	0	17	8
9	1	18	5

(R)

4-in-a-Line

Rules

- Take turns. Pick any two of these numbers to make an $\frac{a}{b}$ fraction.

3	4	5	6	8

- Divide. Mark the decimal name for your fraction on the game board (use x or o).
- Four marks in a line wins.

Game Board

1.3	0.6	0.625	1.25
1.6	0.375	0.83	0.75
0.8	1.2	0.6	1.5
1.6	2.6	0.5	2.0

NCTM – Feb. 1984.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Calculators are limited by an 8-digit display. This creates a problem for determining the repeating period of some fractions. For example, the display on an 8-digit calculator for one-seventh ($\frac{1}{7}$) is 0.1428571. Is the last digit (one) the beginning of the repeating period of more than 8 digits? Students should be taught how to determine the repeating period in examples like this.

   See Journeys in Math 9, p. 83.

2. Two BASIC computer programs follow. The first program will give the decimal equivalent of a fraction entered into the program (to 12 decimal places). The second program can be used to generate a set of decimal equivalents of a given set of fractions (e.g., sevenths: one seventh, two sevenths, three sevenths . . . seven sevenths). This program may be useful in determining the pattern that immerges in the digits, so that a repeating period can be identified. It should be used after using calculators (see #1 above).
   NOTE: The computer will round the twelfth digit in the display. This digit should be ignored when looking for a pattern.

   Program 1

```
  5  REM Decimal equivalents
 10  HOME
 20  PRINT "This program will find decimal"
 30  PRINT "equivalents of common fractions"
 40  PRINT "to 12 decimal places."
 50  INPUT "What is the numerator?";N
 60  INPUT "What is the denominator?";D
 70  F = N / D
 80  PRINT "The fraction, "N;"/"D;" is equal to:"
 90  PRINT N/D
100  PRINT "AGAIN? (Y/N)"
110  INPUT ANS$
120  IF ANS$ = "Y" THEN GOTO 50
130  IF ANS$<>"N" THEN GOTO 100
140  END
```

   Program 2

```
  5  REM DECIMAL EQUIV OF 1/N TO N/N
 20  INPUT "DENOMINATOR?";D
 30  N = 1
 40  N = N + 1
 50  PRINT N / D
 60  IF N < = D - 1 THEN GOTO 40
 70  PRINT "ANOTHER FAMILY OF DENOMINATORS? (Y/N)?"
 80  INPUT ANS$
 90  IF ANS$ = "Y" THEN GOTO 20
100  IF ANS$ < > "N" THEN GOTO 70
110  END
```

OBJECTIVE: 7. Converts rational numbers from decimal form to $\frac{a}{b}$ form (limit: terminating decimals).	Hm 9	JM 9
	76-77	80-81

CLARIFICATION OR EXAMPLE

(Holtmath 9 Teacher's Edition, pp. 76-77.)

Encourage mental computation and check answers using calculators.

ELECTIVE SUGGESTIONS

(E) Change repeating decimals to fractions.
See: Holtmath 9, pp. 76-77.
Journeys in Math 9, pp. 80-83.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Can use a calculator to explore patterns.

OBJECTIVE 8. Computes the square root of whole numbers using estimation and a calculator.	Hm 9	JM 9
	172-177	106-109, 114-115

CLARIFICATION OR EXAMPLE

Encourage students to estimate before calculating.

Start with the idea of squaring. 144 means 12×12. Then go into the opposite operation of square roots.

i.e., $8 \times 8 = 64$

$\therefore \sqrt{64}$ is 8

Establish a table to show squaring and square root are opposite operations.

e.g.,

n	n^2
2	4
3	9

See Holtmath 9 Teacher's Edition , p. 174.

ELECTIVE SUGGESTIONS

(R) Use prime factorization to find square roots.

$$\begin{aligned} 576 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3) \\ &= 24 \times 24 \end{aligned}$$

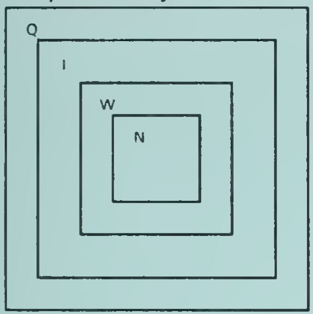
$$\therefore \sqrt{576} = 24$$

OBJECTIVE 9. Demonstrates the relationship among whole numbers, integers and rational numbers.

Hm 9	JM 9
26	27, 36-37, 64-67

CLARIFICATION OR EXAMPLE

The intent is for students to demonstrate the relationship between the number systems. One way this can be demonstrated is pictorially.



Use a fraction tape to illustrate the relationship.

−2	− 1	−10	+ 1	+ 2
	$-\frac{1}{1}$			$\frac{2}{1}$
	− 1.0			2.0

ELECTIVE SUGGESTIONS

Have students complete a chart.

	N	W	I	Q
1				
$\sqrt{3}$				
$\frac{3}{5}$				
−0.125				
π				
3				

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 10.	Understands and uses the following properties (limit: numerical bases and exponents): • $a^x \times a^y = a^{x+y}$ • $a^x \div a^y = a^{x-y}$ • $(a^x)^y = a^{xy}$ • $a^1 = a$ • $a^{-x} = \frac{1}{a^x}$ (limit: a = 10)	Hm 9	JM 9
		154-157, 162, 158-161, 164, 166, 183	96-97, 98-99, 100-101

CLARIFICATION OR EXAMPLE

1. For a concrete approach refer to the Grade 7 and 8 Number Systems and Operations sections of this curriculum guide.

2. Explore and generate exponent laws using patterns.

   e.g., $a^{-x} = \frac{1}{a^x}$

   $5^4 = 625$

   $5^3 = 125$

   $5^2 = 25$

   $5^1 = 5$

   $5^0 = 1$

   $5^{-1} = \frac{1}{5}$

   As the exponent decreases in value, so does the value of the power.

ELECTIVE SUGGESTIONS

(E) After developing the exponent properties with numerical bases and exponents, extend the properties to the general case, using literal bases. The extension of the properties to the general case may not be appropriate for all students. Use concrete examples of literal bases as appropriate.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 11. Writes large and small numbers in scientific notation: (e.g., 0.000 08 = 8×10^{-5}).	Hm 9	JM 9
	170-171	102-103

CLARIFICATION OR EXAMPLE

Discuss examples of very large and small numbers and the need and advantage of using scientific notation to describe these numbers. Students will have been taught about large numbers (positive exponents) in Grade 8. Extend the concept to include small numbers (negative exponents).

ELECTIVE SUGGESTIONS

(R) Multiply by powers of 10 (positive and negative exponents).

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

EXTENDED CONTENT OBJECTIVE: Computes with numbers written in scientific notation (multiplying and dividing).	Hm 9	JM 9

CLARIFICATION OR EXAMPLE

Apply exponent laws to multiplying and dividing numbers written in scientific notation:

e.g., $(8.7 \times 10^{12}) \times (3.3 \times 10^{-8})$

$= (8.7 \times 3.3) \times (10^{12} \times 10^{-8})$ (application of associative property where changing the order of multiplication does not affect the outcome)

$= 28.71 \times 10^{4}$

$= 2.871 \times 10^{5}$

Discuss the limitations of using a calculator or computer to operate with large numbers in standard form (i.e., limited display of digits) and how the "scientific form" helps to overcome the limitations.

ELECTIVE SUGGESTIONS

(E) Develop a strategy for adding or subtracting numbers written in scientific notation.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Using calculators and computers, multiply large numbers and observe the results when the instruments express the answers in scientific notation.

i.e., 3.43501E + 05.

RATIO

and

PROPORTION

9

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 9	JM 9
	208-210	209, 281, 295

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comments: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 37 Problem 9.13; p. 39 Number 3; p. 42 Number 39; p. 45 Number 17.

OBJECTIVE 2. Maintains previously developed skills (understands and constructs ratios, rates and proportions; finds the missing element of a proportion; writes ratios as percents; converts fractions and decimals to percents and percents to fraction and decimal forms; finds missing values in commission, sales tax, and discount situations).	Hm 9	JM 9
	188-199, 200-201, 204, 205	267-269, 272-273, 276-277, 280, 282-283, 288-291

CLARIFICATION OR EXAMPLE

Review the concepts, emphasizing understanding. Calculators should then be used wherever appropriate (e.g., finding commission, sales tax, discounts, conversions between fractions, decimals, percents).

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

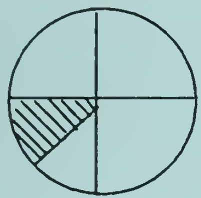
BJECTIVE 3. Converts fractional percents to fraction and decimal forms: e.g., $12\frac{1}{2}\% = \frac{1}{8} = 0.125$	Hm 9	JM 9
	198-199	284-285

LARIFICATION OR EXAMPLE

ncourage the use of calculators to find commission, sales tax and discount.

evelop the concept of fractional percentages at a semi-concrete level before formal development ccurs.

e.g.,



12.5%

se flash cards for drill and practice.

ncourage the use of calculators.

LECTIVE SUGGESTIONS

R) Encourage mental computation with simple fractional percentages.

Have students look for patterns:
e.g.,

$100\% = 1$

$87\frac{1}{2}\% = ?$

$75\% = \frac{3}{4}$

$62\frac{1}{2}\% = ?$

$50\% = \frac{1}{2}$

$37\frac{1}{2}\% = ?$

$25\% = \frac{1}{4}$

$12\frac{1}{2}\% = ?$

NTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations.	Hm 9	JM 9
	200-206, 207	286-287, 290-293

CLARIFICATION OR EXAMPLE

This objective is an extension of Objective #5 (Ratio and Proportion, Grade 8) where students were always given the percent and asked to find a missing element. Finding the percent (given other elements) is subtly different. Solutions will occur in fraction or decimal form and must be converted to percent.

ELECTIVE SUGGESTIONS

(E) Have students determine monthly payments for a car loan. Provide students with information as to: down payment required; current interest rate; length of payment period.

E) Have students investigate the difference in total amount of money payable if a house mortgage is amortized over 15 years, 25 years and 35 years. Amortization tables are required.

(E) With the use of calculators, have students calculate and compare the amount of interest they would earn if a given sum of money was placed in a bank account where interest was compounded semi-annually or monthly.

Discuss different bank accounts and the advantage of one over another.

i.e., Regular savings (interest twice a year)
Daily interest.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. <u>Problem-Solving Project</u>

   Investigate the cost of purchasing a car vs renting a car.
   e.g., payments
   insurance
   maintenance
   gas

2. Compound interest is not always calculated annually. It could be calculated semi-annually, quarterly, monthly, or daily. Find a formula to calculate interest compounded "n" times during a year.
   (From Holtmath 9 Teacher's Edition, p. 209.)

3. "Problem Solving Challenge for Mathematics" p. 50 Number 15.

BJECTIVE 5. Interprets maps and scale drawings.

Hm 9	JM 9
196-197, 214-215	274-275

LARIFICATION OR EXAMPLE

e a class set of atlases to generate a discussion about scale:
What is it?
How is it expressed?
Calculate actual and scale distances.
scuss trips students have taken and use maps to get the actual from the scale distance.

.ECTIVE SUGGESTIONS

) Students are planning a trip in which they rent a car. They want to calculate costs for distance using the scale drawing of a map.

) Review of metric conversions cm to km, cm to m.

ITEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

BJECTIVE 6. Uses a scale to construct drawings, maps or pictures.

Hm 9	JM 9
214-215	

LARIFICATION OR EXAMPLE

k students to draw a cartoon character, a school or company logo, a rock band logo, a map or a plan of ɔuilding, room, etc., on a smaller (or larger) scale.

udents should be taught how to enlarge or reduce with a scale.

ıe criteria for evaluating the project should be clearly defined (e.g., accuracy of scale, proportion, eatness).

.ECTIVE SUGGESTIONS

udents who experience difficulty with this should be encouraged to use <u>grid</u> paper to help them with oportion.

) Give students two drawings, actual and proportion. Have them work out the scale.

) Have the students measure and draw a scale diagram of the classroom.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Have a student use scale drawings in problems such as:

Merv has a garden plot 12 m by 9 m. He wishes to plant vegetable plots 2 m by 9 m. What is the maximum number of plots Merv can plant? How many arrangements are possible?

OBJECTIVE 7. Applies ratio and proportion in practical situations (e.g., uses shadows to find the height of a pole or building; comparative shopping; building a model, computing a test or report card mark based on weighted averages).

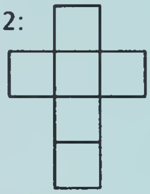
Hm 9	JM 9
190-191, 196-197, 212-215	272-275

CLARIFICATION OR EXAMPLE

Students to develop a project in which they build an actual model from a scale.

e.g. 1: school logo
cars
mascot

e.g. 2:



From this 'net' drawing construct a model that is 6 times as large (1:6).

ELECTIVE SUGGESTIONS

(E) Give students their marks for the term with each of the assigned weightings. Have them calculate their term marks. Compare the results to the computer printout.

(R) Have students check unit prices in their local grocery store.

(R) Determine which item is the better buy by finding the unit prices.

e.g., 3 blank tapes for $5.99 or
2 blank tapes for $4.69.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Have student investigate problems to determine when the most economical buy is not always the best buy.

MEASUREMENT

and

GEOMETRY

9

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 9	JM 9
	242-243	197, 201, 213, 215, 217, 219, 257

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comment: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

"Problem Solving Challenge for Mathematics" p. 35 Problem 9.10; p. 45 Number 9; p. 46 Number 18; p. 46 Number 17.

OBJECTIVE 2. Maintains previously developed skills (linear, area, volume, capacity, and mass units of measure; classification of polygons; perimeter and area of polygons and the circle; volume of a right rectangular prism and cube).	Hm 9	JM 8
	89, 92-95, 98-101, 112-136, 219	125, 159, 193, 196-204, 229

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Problem: You have a 5 L jar and a 3 L jar, both of which are not marked in any way. You may use as much water as you need. Describe how to obtain exactly 4 L of water by filling and emptying the jars.

2. "Problem Solving Challenge for Mathematics" p. 36 Problem 9.11; p. 44 Number 2.

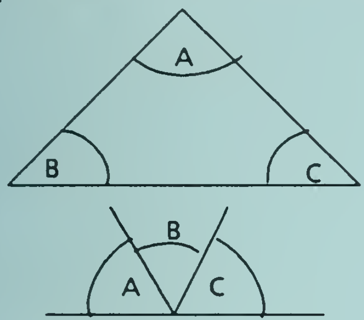
OBJECTIVE 3. Uses concrete manipulatives to determine the sum of the angles in a triangle (180°).

Hm 9	JM 9
	251

CLARIFICATION OR EXAMPLE

Draw and cut out any triangle.

Then cut or tear each angle from the triangle. Place them together to show that the sum of the interior angles equals 180°.



ELECTIVE SUGGESTIONS

(R) Have students measure and record angles of various triangles, then determine the sum for each.

This exercise is an appropriate point to discuss accuracy of measurements.

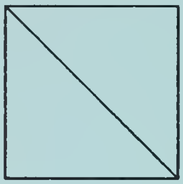
INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Determines the sum of the interior angles in polygons.	Hm 9	JM 9
	232-235	251-253

CLARIFICATION OR EXAMPLE

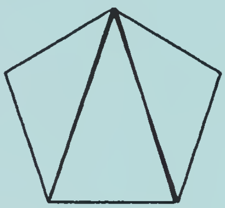
Explore various strategies that could be used to determine the sum of angles in any given polygon. One possibility is to measure each angle with a protractor and then determine the sum. Another strategy is to divide the shape into triangles as shown.

e.g.,



square = 2 triangles
= 2 x 180°
= 360°
∴ sum of interior angles is 360°

e.g.,



pentagon = 3 triangles
= 3 x-----
= ---------
∴ sum of interior angles is -----

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 5. Uses concrete manipulatives to develop the Pythagorean relationship in right triangles.

Hm 9	JM 9

CLARIFICATION OR EXAMPLE

(See Holtmath 9 Teacher's Edition, p. 179, "Alternate Teaching Strategies".)

ELECTIVE SUGGESTIONS

(E) Explore Pythagorean triplets. Students can use a calculator, write a computer program or use a spreadsheet to generate Pythagorean triplets. The triplets can be generated using the expressions:

$$(m^2 - n^2, 2mn, m^2 + n^2)$$

(E) Explore the contention that the products of Pythagorean triplets are always divisible by 60. Does it work? Why or why not?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

See elective suggestions.

OBJECTIVE 6. Applies the Pythagorean relationship to practical situations.

Hm 9	JM 9
178-180, 182, 250	111-114

CLARIFICATION OR EXAMPLE

The Pythagorean theorem lends itself to solving many practical problems. Some strategies that can be reinforced are: drawing a diagram, making a simple problem, developing a code or symbol system, documenting the process, looking back to determine if the solution is reasonable, making and solving other similar problems.

ELECTIVE SUGGESTIONS

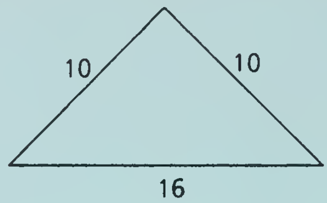
(E) The Pythagorean theorem can be used to find the distance between two points on a grid.

(E) Research the historical development of the Pythagorean theorem.

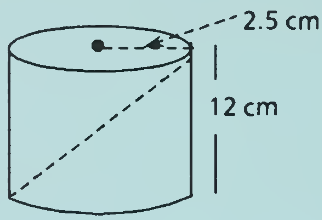
INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Problems

   a) An isosceles triangle with sides of length 10 cm and 16 cm is folded in half. How long is the crease?



   b) Find the length of the longest needle that can be placed in a cylinder that has a radius of 2.5 cm and a height of 12 cm.



2. "Problem Solving Challenge for Mathematics" p. 48 Number 14, 17.

3. One of the powerful uses of computers is to perform repetitive tasks. It is important to note that the task of programming a computer or formatting a spreadsheet requires an understanding of the mathematics concepts involved. Students should be encouraged to write programs, or format and use spreadsheets to perform routine tasks. An example of a BASIC program and a spreadsheet from AppleWorks that can determine the missing side of a right triangle follow. Note that in the spreadsheet, the values of 'A' and 'B' should be entered into cells 'B5' and 'C5'. The formula in cell 'D5' will compute and report the value of 'C'. Variations for finding the values of 'A' and 'B' can also be entered.

   Once a program or a spreadsheet is operational, it can be used for answering routine questions and for problem solving. The emphasis, however, should always be on the understanding of the mathematics concepts and on the problem solving associated with setting up programs or routines.

```
SPREADSHEET

File: PYTHAGORAS

========A=============B===========C===========D==========E====

1:PYTHAGOREAN TRIPLETS
2:
3:
4:ENTER VALUES
5:FOR A AND B             ..........   ..........(B5^2) + (C5^2)^.5)
6:
7:
8:
9:
```

```
5   REM PYTHAGOREAN THEOREM
10  HOME
15  FLASH
20  PRINT "CAPS LOCK DOWN"
30  NORMAL
50  PRINT "This program will find the missing side"
60  PRINT "of a right triangle, given two sides."
80  VTAB (5)
90  PRINT "Identify the UNKNOWN SIDE (A, B or C)"
100 PRINT "where C is the side opposite the right"
110 PRINT "angle (hypotenuse).  C is the longest"
111 PRINT "side and must be larger than A or B!"
120 VTAB (10)
130 INPUT SIDE$
150 IF SIDE$ = "A" THEN GOTO 190
160 IF SIDE$ = "B" THEN GOTO 230
170 IF SIDE$ = "C" THEN GOTO 270
180 IF SIDE$ < > "A" OR SIDE$ < > "B" OR SIDE$ < > "C" THEN GOTO 80
190 INPUT "LENGTH of side C = ";C
200 INPUT"LENGTH of side B = ";B
205 IF B = > C THEN GOTO 350
210 PRINT "The length of side A = "; SQR ((C ^ 2) – (B ^ 2))
220 GOTO 300
230 INPUT "LENGTH of side C = ";C
240 INPUT "LENGTH of side A = ";A
245 IF A = > C THEN GOTO 350
250 PRINT "The length of side B = "; SQR ((C ^ 2) – (A ^ 2))
260 GOTO 300
270 INPUT "LENGTH of side A = ";A
280 INPUT "LENGTH of side B = ";B
290 PRINT "The length of C (hypotenuse) = "; SQR ((A ^2) + (B^2))
295 VTAB (16)
300 PRINT "AGAIN? (Y/N)"
310 INPUT ANS$
320 IF ANS$ = "Y" THEN GOTO 350
330 IF ANS$ < > "N" THEN GOTO 300
340 END
350 HOME
360 GOTO 90
```

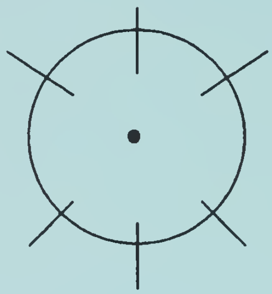
OBJECTIVE 7. Constructs regular polygons using tools such as a computer, ruler, protractor and/or compass.	Hm 9	JM 9
	234-235	91 (Gr. 8)

CLARIFICATION OR EXAMPLE

The emphasis should be on those attributes or characteristics of regular polygons that facilitate their contruction.

For example: To construct a regular hexagon we use the knowledge that the circumference of a circle is approximately 6 times the radius (approximately 3 x d or 6 x r). After constructing a circle (without changing the radius setting of a compass) mark 6 arcs around the circle. Students should note that the last two intersecting arcs don't quite meet on the circle. (The reason is because the circumference is not 6 times the radius but more closely (2 x 3.14) 6.28 times the radius.)

e.g., An inscribed hexagon



When working through a cosntruction, questions like the following should be asked:

a) Why does it work?
b) Is there another way?
c) Can any other polygons be constructed using the same attributes or construction strategy?
d) How can other polygons be constructed? What characteristics would be used? What tools?

ELECTIVE SUGGESTIONS

E) Have students create designs using polygons constructed.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Use a graphics or LOGO program to generate regular polygons. Investigate attributes such as:

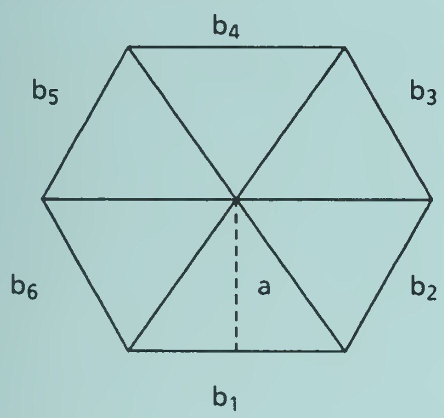
a) measure of angles
b) measure of sides
c) relationship of perimeter to diagonal (diameter).

OBJECTIVE 8. Understands and uses a strategy to determine the area of a regular polygon.

Hm 9	JM 9
92-95, 98, 99	200-201, 205

CLARIFICATION OR EXAMPLE

Build on previous skills (area of triangles, parallelograms, and trapezoids). Expand strategies to find areas of many polygons. For example, a hexagon can be divided into 6 triangles.



Total area = sum of 6 triangles.

$A = \frac{1}{2}a(b_1 + b_2 + \ldots + b_6)$

$A = \frac{1}{2}ap$

$A = \frac{1}{2}$ (apothem) (perimeter)

Note:

$p = ns$
(perimeter = no. of sides + length of side)

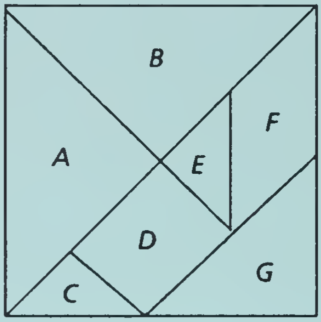
ELECTIVE SUGGESTIONS

(R) Use cutout pieces to make shapes.

(Arithmetic Teacher, Vol. 31, #5, January 1984, pp. 28-32.)*

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. The figure shows a square separated into the seven pieces of an ancient puzzle called the tangram. If the area of the entire square is one square unit, what is the area of each of the seven tangram pieces?



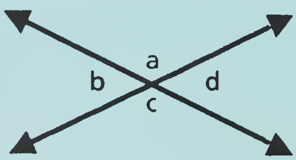
"Creative Problem Solving in School Mathematics" Houghton Mifflin, p. 134.[1]

2. See: "Problem Solving Challenge for Mathematics" p. 14, Problem 5.

OBJECTIVE 9. Identifies pairs of angles: (supplementary, complementary, adjacent and opposite).	**Hm 9**	**JM 9**

CLARIFICATION OR EXAMPLE

This is the appropriate time to reinforce the use of a ruler and a protractor.
Have students draw two intersecting lines and then measure the resulting angles. They will then discover the resulting relationships.



Angles:

$a + b = 180°$
$b + c = 180°$
$c + d = 180°$
$d + a = 180°$
$a = c$
$b = d$

* Reprinted with permission from National Council of Teachers of Mathematics.

[1] Reprinted with permission from Houghton Mifflin Company.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 10.	Hm 9	JM 9
Uses a compass and a straightedge to construct: • **a congruent segment** • **a congruent angle** • **a perpendicular bisector of a segment** • **a bisector of an angle** • **a perpendicular to a line** • **angles of 90°, 45°, 60°, and 30°.**	220-221, 222-223, 375	230-231, 232, 234-235

CLARIFICATION OR EXAMPLE

(Holtmath 9 Teacher's Edition, pp. 220-229.)

Angle construction is a direct extension of the following angles:

90°– a perpendicular
45°– bisector of a 90° angle
60°– construction of an equilateral triangle
30°– bisector of a 60°angle

(Holtmath 9 Teacher's Edition, pp. 220-225.)

ELECTIVE SUGGESTIONS

(E) Explore other constructions, such as constructing a perpendicular to a point not on a line, finding an altitude of a triangle.

(E/R) Use other tools to perform the constructions (e.g., Mira, LOGO).

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 11. Given nets, constructs right prisms.	Hm 9	JM 9
	108-109, 240-241	206-207

CLARIFICATION OR EXAMPLE

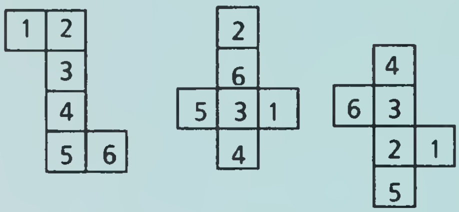
Given a net of a right rectangular prism, have students fold and paste to make a box. Explore its construction, making note of its characteristics (faces, edges, vertices). Expand this to students exploring other right prisms (for example, a cube, a triangle prism, a pentagonal prism) and their construction.

ELECTIVE SUGGESTIONS

1. (E) Show students examples of other figures (tetrahedron, dodecahedron, and so on). Explore the nets for these.

2. (R) Have a collection of boxes (cracker boxes, cereal boxes, matchboxes, etc.). Students can then dismantle the boxes and examine the nets. Encourage them to look for the similarities and differences.

   Problem:



   Each of the above patterns can be folded to form a cube. Which two cubes will look the same?

3. (E) String art on an octahedron constructed of drinking straws.
   (Arithmetic Teacher, Vol. 34, #3, November 1986, pp. 30-33.)*

4. (E) Construct right pyramids from nets. Compare the characteristics of pyramids and prisms.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

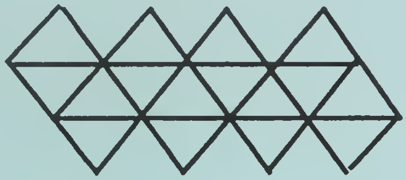
1. Problem Solving: Discuss how to produce nets of right prisms or cylinders, given specific conditions.

   e.g., You have a piece of paper (loose-leaf size). Construct a net for a cylinder that has a 5 cm radius. Problems of this type have more than one solution since the condition not specified is the height.

* Reprinted with permission from National Council of Teachers of Mathematics.

2. Have a three-dimensional model of an icosahedron available for students to examine. Give everyone a pattern for constructing an icosahedron. Ask each student to colour a pattern such that, after the figure has been cut out and taped together, no two adjacent faces (faces that share an edge) will be the same colour. Furthermore, everyone should attempt to use the fewest number of colours. Finally, have the students cut out their patterns and tape them together to check their solutions.

(diagram)



(Arithmetic Teacher, Vol. 31, #7, March 1984, p. 43.)*

3. "The Super Factory" Sunburst Communications.

OBJECTIVE 12. Classifies right prisms and cylinders.	Hm 9	JM 9
	108-109	206-207

CLARIFICATION OR EXAMPLE

From exploration of constructing nets, students should intuitively know the characteristics of various right prisms and cylinders. Classification should simply be a natural extension and formalization of the attributes.

i.e., rectangular right prism
triangular right prism
square right prism (cube)
right circular cylinder

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

* Reprinted with permission from National Council of Teachers of Mathematics.

EXTENDED CONTENT	Hm 9	JM 9
OBJECTIVE: Classifies right pyramids.		

CLARIFICATION OR EXAMPLE

Have models of various pyramids and discuss common attributes and differences. Use words such as faces, bases, edges and vertices.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

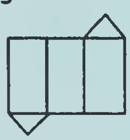
OBJECTIVE 13.	Hm 9	JM 9
Understands and uses a strategy for finding the surface area of any right prism or cylinder.	108-109	208-211, 218

CLARIFICATION OR EXAMPLE

Review the concept of area (as a count of squares) and the role that formulas play in determining area. (Formulas provide an indirect strategy for counting squares.) Note that the strategy for determining surface area of a prism is simply a matter of using several formulas that describe the number of squares within the different shapes that make up its surface. The development of the concept of surface area should be a logical extension of the students' investigations with nets.

Some students may choose to determine surface area by breaking the prism down into a net and then computing the sum of the areas of the different polygons. Others may be capable of generalizing the strategy and developing a formula. Either strategy should be acceptable.

e.g.,



A triangular prism is made up of 3 rectangles and 2 triangles.

ELECTIVE SUGGESTIONS

(R) Using models of prisms and cylinders, take them apart and explore the area of each piece.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Problem Solving: Strategies can be developed and expanded upon through problem solving. For example "Susan built a toy chest for her little brother. The dimensions of the toy chest are 95 cm by 40 cm by 50 cm. Susan wanted to paint the outside (including the bottom) with enamel paint. What is the total surface to be painted?"

2. "Problem Solving Challenge for Mathematics" p. 47 Number 9.

OBJECTIVE 14. Understands and uses a strategy for finding the volume of any right prism or cylinder.	Hm 9	JM 9
	101-103, 112, 114, 136	212-215, 218

CLARIFICATION OR EXAMPLE

Review the concept of volume (as a count of cubes) and the role that formulas play in determining volume. (Formulas provide an indirect strategy for counting cubes.) The development of a strategy for determining the volume of a prism should be tied to the concept of volume. Note that the number of cubes in one layer will correspond to the area of the base of the prism. Note also, that the number of layers correspond to the height of the prism. The product of the area of the base and the height can indirectly determine (count) the number of unit cubes that can be placed within a prism (volume).

Begin the development with right rectangular prisms and extend the concepts to other right prisms.

ELECTIVE SUGGESTIONS

(E) Write a computer program that can be used to calculate the volume of right prisms and cylinders.

(E) Explore how changing one characteristic of a prism or cylinder changes its volume. For example: A cylinder with a height of 6 cm and a radius of 5 cm is changed to a cylinder with a radius of 10 cm. What happens to the volume?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Problem Solving – Apply strategies to solving non-routine problems.

   e.g., A solid cube is painted red on all sides. The cube is then cut into 27 equal smaller cubes. How many of the smaller cubes have red paint on only 2 sides?" Problem Solving Challenge for Mathematics" (Alberta Education, 1985) p. 47.

2. "Problem Solving Challenge for Mathematics" p. 44-45 Number 76; p. 47 Number 9.

3. If a glass of water resting on a table is tilted slightly, does the height of the water (from table level) change? If it does change, how? Why or why not?

DATA

MANAGEMENT

9

Whenever possible, the objectives of this strand may be integrated throughout the program. The effect is to make the objectives more meaningful than when taught in isolation.

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 9	JM 9

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing) be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comment: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Maintains previously developed skills (understands purpose, use and misuse of statistics; biases in surveys; represents data in the form of pictographs, bar graphs, line graphs, circle graphs).	Hm 9	JM 9
	334-337, 348-351	334-337, 342-348

CLARIFICATION OR EXAMPLE

See Grade 8 Data Management, Objectives #2, 4.

ELECTIVE SUGGESTIONS

(E) Watch a newscast on TV to determine when, where, how and why statistics are used. Discuss their accuracy and timeliness.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problem-solving strategies and skills can be a natural outgrowth of a class survey.

e.g., collect, organize and interpret data;
make decisions;
estimate;
predict;
draw inferences.

OBJECTIVE 3. Analyzes and interprets arguments or conclusions based on statistical information.	Hm 9	JM 9
	348-351	346-348

CLARIFICATION OR EXAMPLE

Have students collect articles that report statistical information from magazines and newspapers.

Discuss the articles using the following questions as a guideline.

- Is the result typical of the population?
- Is the survey large enough to make prediction?
- Are there any noticeable scale variations (e.g., bar graphs without a zero value)?
- Are there any misrepresented data, incomplete data, or misleading data?

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range.	Hm 9	JM 9
	342-343	352-353

CLARIFICATION OR EXAMPLE

In using quiz scores (such as 18, 28, 27, 19, 24, 40, 27, 35, 27, 39, 28, 40, 25, 36, 33, 27, 27) determine the measures of central tendency:

a) mean (average)
b) range of values
c) median (middle value)
d) mode (most frequent value).

Discuss when and why each of these measures may be used.

See Holtmath 9 Teacher's Edition, pp. 342-343.

ELECTIVE SUGGESTIONS

(E) Compare the measures of central tendency (mean, mode, median, range). Determine which is most appropriate within its set of data.

(E) Find the effect on mean, median, mode and range if:

a) a very high score is added
b) the frequency of the lowest value is increased several times
c) more than one value is most frequent.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 8, Program 1 "Three M's".

2. Students can design a simple computer program to calculate the mean and possibly the mode, median and range.

OBJECTIVE 5. Distinguishes between a percent and a percentile.	Hm 9	JM 9

CLARIFICATION OR EXAMPLE

The intent of this objective is to convey to students, the understanding that there is a difference between the terms 'percent' and 'percentile' and that the two terms cannot be used interchangeably. Percent is a fraction, percentile is a ranking. The development of this objective should not exceed this understanding.

Real-life situations and practical examples can be used to explain the difference.
e.g., 1. Compare percents of test scores to a ranking of the same scores.
2. A health nurse or doctor tells a mother that her baby, weighing 8 kg, is in the 20th percentile of mass for babies that age.

ELECTIVE SUGGESTIONS

(R) Develop a chart based on test scores. Approximate the ranking (percentile) that the various scores would have.

(E) Conduct a survey on height or weight of people and construct a table. Use the table to determine in what percentile people who were not surveyed would be.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 6. Conducts a survey or poll using correct sampling techniques and reports results using an appropriate table, chart and/or graph.	Hm 9	JM 9
	344-347	340-341, 344-345

CLARIFICATION OR EXAMPLE

Decide what would be a typical sample group. Conduct a survey and choose the best form for displaying the results. Did you use an unlimited response, such as, what is your favourite sweater colour?

ELECTIVE SUGGESTIONS

(R) Use a restricted choice survey (e.g., What is your favourite dessert – strawberry jello; banana, butterscotch pudding; chocolate ice cream?) and decide on the target group.

(E) Conduct a glasses survey: Who wears eyeglasses, contacts, sunglasses, combinations, and who doesn't?

How would one answer the question of who wears glasses and misuse the information?

Was the survey based on age groups, income groups, etc?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 7. Understands and uses the term probability.	Hm 9	JM 9
	356-357	354-357

CLARIFICATION OR EXAMPLE

When an outcome of an event is not known in advance the words 'chance' or 'probability' are used to describe the likelihood of the desired outcome occurring. If the chance or probability of the school team winning the chamionship is high, then the desired outcome is 'highly likely'. If the chance or probability is low, then a championship for the school is described as 'unlikely'. Many events (winning a game, weather, passing a test, going to a dance, winning an election etc.) are reported in terms of chance or probability and many decisions (agriculture, business, industry, medicine, politics, personal, etc.) are made on the basis of a determined probability. The goal of mathematics is to assign numbers to events and then use these numbers to describe chance or probability.

When discussing probability with students, note that the "certainty" of an event can range from impossible to absolute.

e.g., A person can hold his breath for one hour.
The sun will rise tomorrow morning.

Extend the discussion to include many events whose certainty will fall between impossible and certain.

e.g. Winning a coin toss; winning a game; winning a lottery

The prediction of an occurrence must be more systematic than a dependence on feeling or emotion. In mathematics, numbers are assigned to the desired outcomes and to the total possible outcomes. The ratio of the 'number of actual or desired outcomes' to 'the total number of possible outcomes' is called probability. For example, in a coin toss, the number of heads (the desired outcome) is one; the total number of possible outcomes is two (head and tail). Therefore, the probability of a head is $\frac{1}{2}$.

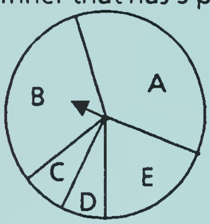
This discussion should lead to an examination of techniques that may be used to determine probability (objective 8).

ELECTIVE SUGGESTIONS

(E) Have students examine lotteries and determine the chances of winning.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. Students design a spinner that has 5 possible outcomes but the sectors are not equal.



Students estimate the probabilities of each outcome. Then they take several spins to see how close they came to the actual probability.

From Journeys in Math 7 TRM.

2. MAC 7, Program 13 "Hit or Miss".

OBJECTIVE 8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer).	Hm 9	JM 9
	358-359	358-363

CLARIFICATION OR EXAMPLE

Students should be aware that there are a number of strategies for predicting an outcome. Using past records, conducting a survey, using statistical information, performing experiments and simulating events are strategies for predicting chance.

The intent of this objective is to provide students with a practical and concrete experience in determining a probability. Ask students to predict the chance of an event before conducting an experiment or activity. Begin with a simple situation like determining the probability of pulling a black or red chip out of a bag containing only two colours. (This experiment can simulate pulling socks (two different colours) out of a drawer in a dark room.

Coin tosses can be used to simulate an event. The result will be an experimental probability of the occurrence of the event.

Example: What are the chances of two children in a family being a boy and girl?

The chance of a boy (or girl) in a single event (birth) is $\frac{1}{2}$. Since the chance of tossing a head or tail is the same, a coin toss can be used to simulate the birth of a girl (or boy). Decide on the representation (e.g., girl – heads, boy – tails) and have each student toss a coin twice (representing the two children in the family). Collect and record all the results on the board and count the number of favourable outcomes; a head and a tail representing a girl and boy. The probability of two children in a family being a boy and girl is the ratio of favourable outcomes to the total number of observations (students).

- Does the probability match the students' prediction?
- Does a survey of two-child families confirm the result?
- What are the chances of a three-child family having two girls and a boy?

ELECTIVE SUGGESTIONS

(E) Other strategies for predicting probabilities may be presented. Past performance or records, or surveys may be used to do a project that will determine the probability of the occurrence of a local event (a school election or local plebiscite, rain or snow on an eventful day (parade, rodeo, graduation, carnival), etc.).

(E) Investigate the probability of winning a lottery.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 8, Program 15 "Probable Urnings".

2. Computer programs that will simulate coin tosses are available. These programs are useful because they can simulate many more tosses in a shorter period of time, than can actually be done in a classroom.

ALGEBRA

9

OBJECTIVE 1. Applies and practises problem-solving skills in new situations.	Hm 9	JM 9
	134-135	159, 176, 177

CLARIFICATION OR EXAMPLE

The intent of this objective is that every new situation (especially if it is perplexing), be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.

See comment: Objective 1, Number Systems and Operations.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 2. Maintains previously developed skills (variables; like terms, evaluation of expressions; solving equations; generating and plotting ordered pairs from a given relation).	Hm 9	JM 9
	12-19, 244-245	158-167, 304-309

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Simple graphing programs can be used to generate ordered pairs and plot these pairs on the screen.

OBJECTIVE 3. Uses formal procedures to solve equations (using all forms of rationals) of the form: $x + a = b$, $ax = b$, $ax + b = c$, $ax + bx = c$, $\frac{x}{a} = \frac{b}{c}$, $ax + b = cx$, $a(x + b) = c$, and $ax + b = cx + d$	Hm 9	JM 9
	124-129	164-171

CLARIFICATION OR EXAMPLE

Students in Grade 8 will have used formal procedures to solve equations of the form $x + a = b$, $ax = b$, $ax + b = c$, $ax + b = c$ and $\frac{x}{a} = \frac{b}{c}$ (limit: positive rationale and integers). The Grade 9 program adds three new forms and removed the limit.

In the form $a(x + b) = c$, the distributive property should be emphasized.

For all forms, use addition, subtraction, multiplication or division of like quantities on both sides of the equality to form equivalent sentences.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 4. Verifies solutions to equations.	Hm 9	JM 9
	124-129	164-170

CLARIFICATION OR EXAMPLE

It is assumed that students can solve equations formally.

They should be encouraged to substitute the answer in the original equation and decide if the answer is correct
(see Grade 8 Algebra, Objective #5).

ELECTIVE SUGGESTIONS

(R) Use manipulatives such as a balance of scale to reinforce the concept of equality.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Use of a calculator is encouraged.

OBJECTIVE 5. Manipulates a given formula to change the subject of the formula:

e.g., given $x = \frac{f}{w}$ then $w = \frac{f}{x}$

Hm 9	JM 9
140-141	172-173, 278-279, 315 (#11)

CLARIFICATION OR EXAMPLE

The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., $d = rt$ is used to find distance, rate or time). The result is that little transfer occurs to knowing how to use other formulas in mathematics and other disciplines.

A much more holistic view of formulas must be taken. In Grade 8, the outcome of objective #6 (Algebra) is that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used) and will know how to substitute and solve for missing elements.

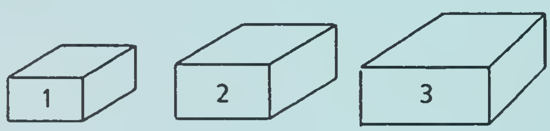
The intent of this objective is to provide students with an alternative strategy for finding the missing value in a formula, especially in situations for which the required missing element is always the same.

For example, if students were required to find the principals in a number of loans, manipulating the formula first (using $p = i/rt$) is much more convenient than using the form $i = prt$ (and then having to substitute and isolate the p). Review the relationships among the variables in any formula (real or contrived) and demonstrate that the techniques for isolating a particular variable are the same whether the variable has been replaced by a number or not.

A formula that can be practically verified is $D = \frac{M}{V}$

where D means Density, M means Mass, V means Volume

Have the students weigh and measure the volume of wooden blocks of different sizes (but same type of wood).



	M	V	$D = \frac{M}{V}$
1			
2			
3			
4			
5			
6			

When the chart is filled, verify the values using $M = DV$ or $D = \frac{M}{V}$

ELECTIVE SUGGESTIONS

(E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc.
e.g., What happens to A, in $A = \pi r^2$, if r is doubled? tripled?

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Computers and calculators may be used to check some of the values on the chart.

Use a spreadsheet to assess effects of changing values in a formula.

e.g.,

```
==========A==========B======C======D======E======F======G====
1:        FORMULAS
2:=====================
3:
4:Radius of Circle         1         2        4         8        12
5:
6:Area of Circle        3.14     12.56    50.24    200.96    452.16
7:                 (3.14*B4*B4)
8:
9:
```

Each of the cells (B6 – F6) contains the formula for computing the area of a circle. When the value of 'r' is placed in cells B4 – F4, the area is computed and displayed in cells B6-F6.

Students should be encouraged to format and use a spreadsheet to investigate the effect on A of:
- doubling r
- halving r
- tripling r
- etc.

Any formula can be investigated in this manner.

OBJECTIVE 6. Finds a missing element of a formula through manipulation.

Hm 9	JM 9
140-141	172-173

CLARIFICATION OR EXAMPLE

This objective is an extension of the previous objective (#5). The development of objective #5 should be extended to include actual computations of values.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 7. Solves inequalities of the form $x + a \geq b$ and $cx \leq d$ (c is positive; direction of inequalities may vary).

Hm 9	JM 9
143-147	178-179

CLARIFICATION OR EXAMPLE

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

Problems in which a maximum or minimum value may be explored.

OBJECTIVE 8. Verifies solutions to inequalities.	Hm 9	JM 9
	145-147	178-179

CLARIFICATION OR EXAMPLE

Only the limit of the solution to an inequality can be verified. For example; for $3x > 9$, $x>3$, the verification can only determine if 3 is the correct limit (or boundary) of the solution. It will not verify all the values of the solution.

e.g., Verify that for $3x > 9$, $x>3$.

Verification:

$$3x = 9$$
$$3(3) = 9$$
$$9 = 9$$

If the solution set was given as $x < 3$, the verification would still work although $x < 3$ does not correctly describe the solution set for x.

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 9. Graphs solutions to inequalities on a number line.	Hm 9	JM 9
	46-47	180-181

CLARIFICATION OR EXAMPLE

Reviews the meaning of the inequality symbols $<$ and $>$ and the combined symbols of $\leq$ and $\geq$. A method for plotting on a number line should be decided upon; i.e., "is more than" is the same as $>$, and can use an open dot ($\circ\!\longrightarrow$) whereas "is less than or equal to" is the same as $\leq$, and can use a solid dot. ($\longleftarrow\!\bullet$)

ELECTIVE SUGGESTIONS

Draw graphs in which the values of x and y are between certain limits (domain and range respectively). Have the students graph the domain and range on a number line.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

OBJECTIVE 10. Given a set of ordered pairs or a table of values, writes the function that determines the relation (limit: linear relations).

Hm 9	JM 9
246-249, 304-311	310-317, 322-327

CLARIFICATION OR EXAMPLE

THE GRAPHING COMPONENT OF THE ALGEBRA STRAND SHOULD BE DEVELOPED (AT ALL THREE GRADE LEVELS) WITH THE FOLLOWING CONTEXT IN MIND:

Numbers relate to each other. The emphasis in this program is placed on the understanding of algebra as a generalization of the relationships and patterns of pairs of numbers. Generally speaking, early elementary school mathematics deals with single numbers and their recognition and operations. The focus of secondary school mathematics shifts to 'pairs of numbers' and the relationships that exist between those pairs. Coordinate geometry (graphing), the use of formulas and eventually the study of trigonometric functions, polynomial functions, logarithmic functions etc., become central to the secondary program.

Students should learn that the relationships occur naturally and are pervasive. They should feel comfortable in describing them. By the end of Grade 9, students should be able identify the relations that are functions.

Relations are defined as the one-to-one correspondence that exists between two elements or two sets of data. Associating a name with an object, completing a 'times' or 'add' table, comparing the height to weight of people, or comparing the area of a circle to its radius, are all relations.

Functions are those relations where the value of the first element determines the value of the second. Given the first element, only one possible pair of numbers can satisfy the condition of the function. The value of the second element "depends" on the value of the first element.

Example 1: When given the radius of a circle, only one value can describe the area. Area is a function of the radius.

Example 2: While the height of a person is related to weight, the relationship is **not** a functional one. A unique pair of numbers cannot describe the relationship (i.e., a person with a given height will not necessarily be a specific weight).

Graphs are pictoral representations of relations. Two types of skills are required for graphing. The first is the **technical skill** of choosing a scale, plotting points and drawing the lines (or curves). The second is the **interpretive skill** that contains elements such as increase, decrease, maximum, minimum, rate of change and slope. Because graphs contain a great deal of information in a relatively small amount of space, the interpretive skills are generally more difficult to deal with and are often neglected.

The development of graphing knowledge and skills is distributed as follows:

Grade 7 – technical skills (plotting points on a coordinate plane, choosing a scale, and drawing the graph).

Grade 8 – technical/interpretive skills (constructing a table of values that identify a function and graphing the relation.)

Grade 9 – interpretive skills (given a graph or table of values, identify the function). (Limited to linear relations.)

Interpretive skills must be developed at **an intuitive level**. Whenever graphs are discussed, the concepts of increase, decrease, intercepts, maximum, minimum, rate of change and slope should also be discussed as appropriate to the graph. Except for the word 'function', the use of the terminology must be de–emphasized. Students should be encouraged to draw or interpret graphs that describe many different types of situations. (Some may not even have, or need numerical information.)

Examples: (1) Sketch a graph that would describe (over time) the temperature of a glass of water just after an ice cube has been dropped into it. (2) Sketch a graph that would describe the speed of a race car on a given track. (Sketch the track on the board or an overhead with several curves, corners, etc.)

Discuss the increases, decreases, intercepts, maximums, minimums, recurring patterns etc., of the graphs without emphasizing terminology. Present a graph to the students (e.g., temperature of bath water vs time) and ask them to describe what they think happened.

ELECTIVE SUGGESTIONS

Have students make up their own tables from a relation. Exchange tables for others to determine the relation form.

(E) Explore how changing a or b in a linear relation of the form $y = ax + b$ changes the graph. Students should be encouraged to predict the quandrants and (intuitively) the slope of the graph.

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

1. MAC 7, Program 15 "Connect the Points".
2. MAC 8, Program 14 "Equation Darts".
3. Explore problems in which patterns develop, using logic or reasoning, generalities solutions.
4. (E) Solve simultaneous linear relations by graphing and determining the point of intersection.

EXTENDED CONTENT

OBJECTIVE: **Knows the terms associated with polynomials: monomial, binomial, trinomial, degree, numerical and literal coefficient.**

Hm 9	JM 9
256-257	130-131

CLARIFICATION OR EXAMPLE

(Holtmath 9 Teacher's Edition, pp. 256-257.)

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

EXTENDED CONTENT

OBJECTIVE: **Finds the sum and difference of monomials.**

Hm 9	JM 9
258-259	128-129

CLARIFICATION OR EXAMPLE

Use manipulatives in the form of squares and rectangles to develop the concept of "like" terms. Squares and rectangles can be used to represent terms. Congruent shapes would represent like terms.

e.g., □ + ▭ + ▭ + ▭ + □ = 2 □ + 3 ▭

Finally, use $3x$ and $4x$ as examples of like terms and $3x$ and $5y$ as unlike terms. Conclude that to add or subtract like terms one must add or subtract the numerical coefficients. Remember, to subtract, add the opposite.

$$4x - (^-2x)$$
$$= 4x + 2x$$
$$= 6x$$

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

EXTENDED CONTENT	Hm 9	JM 9
OBJECTIVE: Finds the product and quotient of monomials.	262-263, 272-273	132-133, 138-139

CLARIFICATION OR EXAMPLE

(MULTIPLYING MONOMIALS)

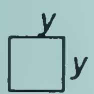
Give some numerical examples of products of the same factors, e.g.,

$4 \times 4 = 4^2$
$4 \times 4 \times 4 = 4^3$
$4 \times 4 \times 4 \times 4 = 4^4$

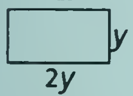
Use the same type of examples with letters instead of numbers

$y \times y = y^2$
$y \times y \times y = y^3$
$y \times y \times y \times y = y^4$

Introduce the idea that the area of a rectangle is the product of two factors (or sides)
e.g., $(2) \times (3) = 6$ square units



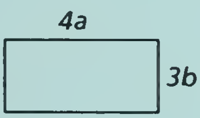
$(y)(y) = y^2$



$(2y)(y) = 2y^2$

Manipulatives to be used for the area concept may be cardboard cut into rectangular shapes.
After this, introduce abstract examples.
e.g.,
a. 5×4
b. $2x \times 3$
c. $(4x) \times (3x)$

Using a rectangle, note the calculation of area using sides of $4a$ and $3b$.
To multiply two or more monomials, multiply the numerical coefficients and multiply the literal coefficients.
e.g., $(4a)(3b) = 12ab$



Using examples with numbers (i.e, $2^5 \div 2^2$) give the rule for division of powers with the same base. To divide monomials, divide the numerical coefficients and subtract the exponents on the same bases.

i.e., $2^5 \div 2^2 = 2^{5-2} = 2^3$

$x^5 \div x^2 = x^{5-2} = x^3$

ELECTIVE SUGGESTIONS

INTEGRATION OF PROBLEM SOLVING AND TECHNOLOGY

APPENDIX A

EVALUATION

RATIONALE

> "A common form of assessment is testing for the purpose of assigning grades. But assessment should be conceived of as a much broader and basic task than just testing and grading. Its basic purpose is to determine what and how students think about mathematics. Assessment should involve the biography of students' learning as well as the continual impact of the instructional program. Such assessment should provide the basis for improving the quality of instruction. Indeed, assessment has no raison d'être unless it is clear how assessment can and will improve instruction."

Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics
"Curriculum and Evaluation Standards for School Mathematics" (p. 139)*

The Junior High School Mathematics Program emphasizes the understanding of concepts and relationships, problem solving and application. The assessment of understanding and problem solving must go beyond determining the percent of correct responses on a test based on mathematical facts. It should also provide information about how individual students approach doing mathematics, the level of understanding students have of concepts, and their ability to apply mathematics in new situations. The role of assessment should be to provide feedback and evidence of progress toward desired instructional goals. A singular assessment technique cannot provide such evidence.

Some assessment strategies follow. These strategies are not intended to be discrete and may be used with other strategies. For example, a checklist may be used to document desirable problem-solving behaviour in a classroom or to guide discussion and evaluate performance in an interview. The applications of these strategies should not be limited to the noted suggestions.

EVALUATION STRATEGIES

Strategy

Interview A planned interview with a student or a group of students is an effective technique for assessing knowledge, understanding, thinking style or attitude; for assessing communication skills (e.g., verbal vs non-verbal ability); and for learning about personal interests. A well-conducted interview will also give students an opportunity to reflect on their own learning.

Interviews should have a definite purpose and both the teacher and the students should be aware of that purpose. The interviews must be planned in advance. Teachers should be prepared to ask additional leading or key questions to guide discussion, to probe for understanding and/or to correct misconceptions.

*Reprinted with permission from National Council of Teachers of Mathematics.

A one-on-one interview, conducted with a single student, may be used to assess individual students while small group interviews may be used for assessing instructional effectiveness and for consolidating concepts and skills.

Observation Teachers observe all the time. When observations are documented their effectiveness as an evaluation strategy increases immensely. Documented observations often provide the raw data required for analysis and diagnosis, and provide the basis on which to make remediation or enrichment decisions.

Unlike the interview, observation is a passive strategy. The teacher observes students at work, looking for specific behaviours or outcomes. Documentation may occur in the form of anecdotal records or checklists. Some elements that may be monitored are understanding of concepts or generalizations (e.g., when using manipulatives or in group discussions), enthusiasm, willingness to participate or share ideas (attitudes), perseverance and independence.

Checklist A checklist is a documentation strategy and is used conjunctively with other evaluation strategies. Checklists can easily be created and customized to meet many different needs and situations. Generally, a matrix is created, listing indicators of desirable behaviours or outcomes on one side, and listing ratings, skill levels or evaluative comments along another side. As teachers note a particular behaviour, they need only check the appropriate column that evaluates or rates that behaviour.

Checklists lend themselves very well to documenting such elements of the program as understanding of concepts using manipulatives; mastery of knowledge, skills or objectives; work habits; organizational skills; problem-solving strategies; and cooperative skills.

Anecdotal Records Anecdotal records refer to the spontaneous documentation of notable behaviour, effort, achievement, attitudes, changes in performance, social skills, communication skills etc. Records may be kept in such locations as a daily or weekly diary, individual student files, a specific location in the marks record book, or in a common file that serves as a collection of short dated notes.

Anecdotal records provide specific and dated information that can form the basis for conclusions and assessments. These records often prove invaluable in clarifying assessments and add credibility to observations and recommendations being offered in student, parent and/or teacher meetings.

Written Assessments Traditional multiple choice and open-ended paper-and-pencil assessments fall into this category.

Two other types of written assessments should be used. One is project writing where students may report on a particular interest or research. The evaluative technique and grading system used in this case is not unlike the one used in language arts or social studies.

The second is diagnostic writing. Writing assignments that respond to specific mathematics questions, in an expressive writing style, have proven successful as a diagnostic tool in mathematics. Written responses often force students to examine their own understanding of concepts and will communicate to teachers how much students really know about a concept. Written responses also provide insight to how much understanding students have and how they think. For example, students may be asked to give a non-mathematical example of the inverse rule ($a - {}^{+}b = a + {}^{-}b$).

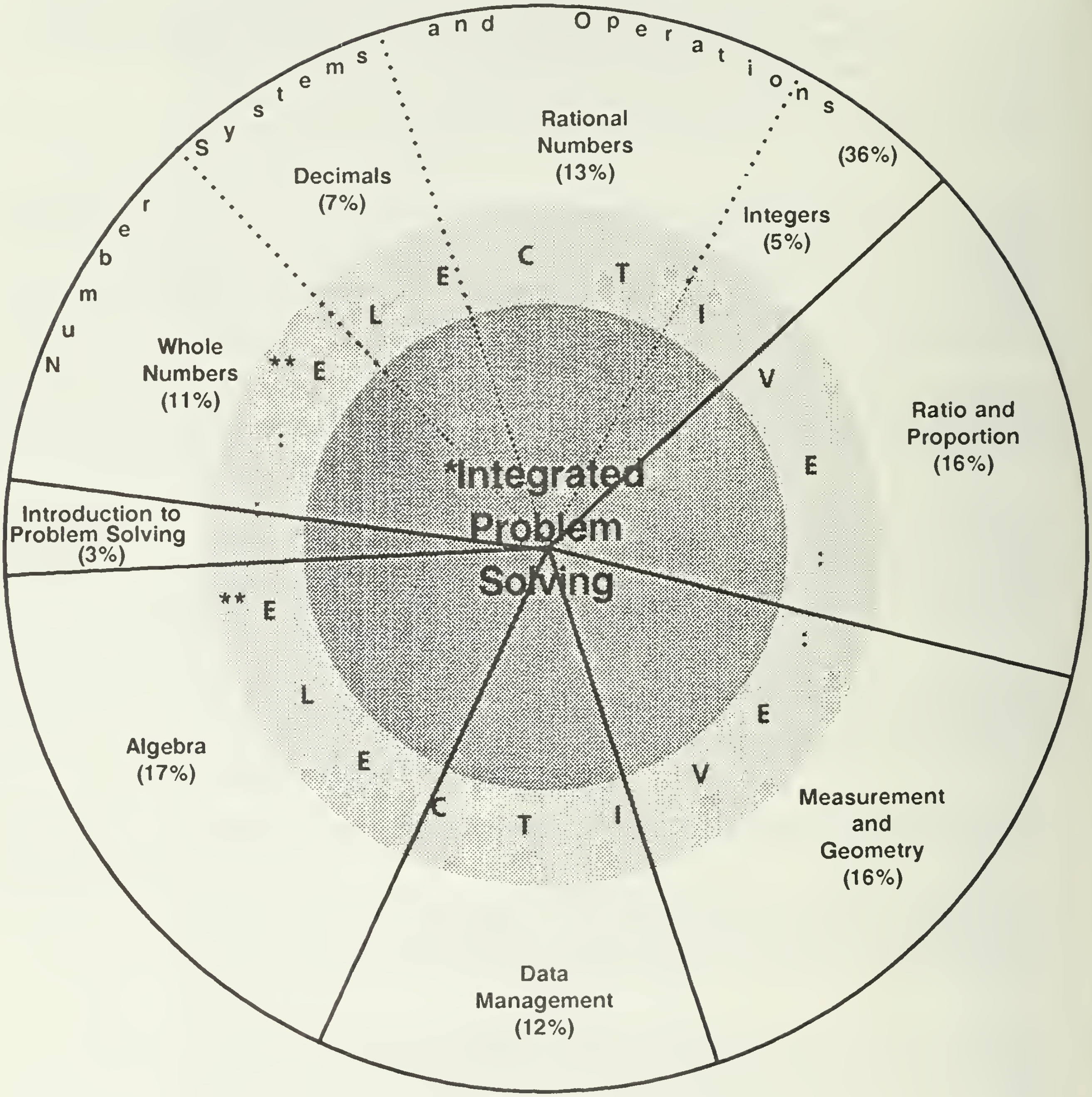
The response will give teachers different information about student understanding than will ten mathematics questions like $3 - 4 =$ __.

Written assignments should be short (one paragraph) and written in an expressive writing style. They should not be graded (for a mark) but, rather, should be assessed for understanding (diagnosis). Appropriate remedial or enrichment actions may be determined, based on students' understanding of a given concept. Students may be awarded bonus points based on their effort and presentation (e.g., 0 for no effort; 1 for mediocre attempt, even with little or no understanding). Written responses can be kept in a student diary or logbook, and may be assigned on a regular basis (once or twice a week), or in lieu of a regular quiz.

APPENDIX B

SUGGESTED TIME ALLOCATIONS

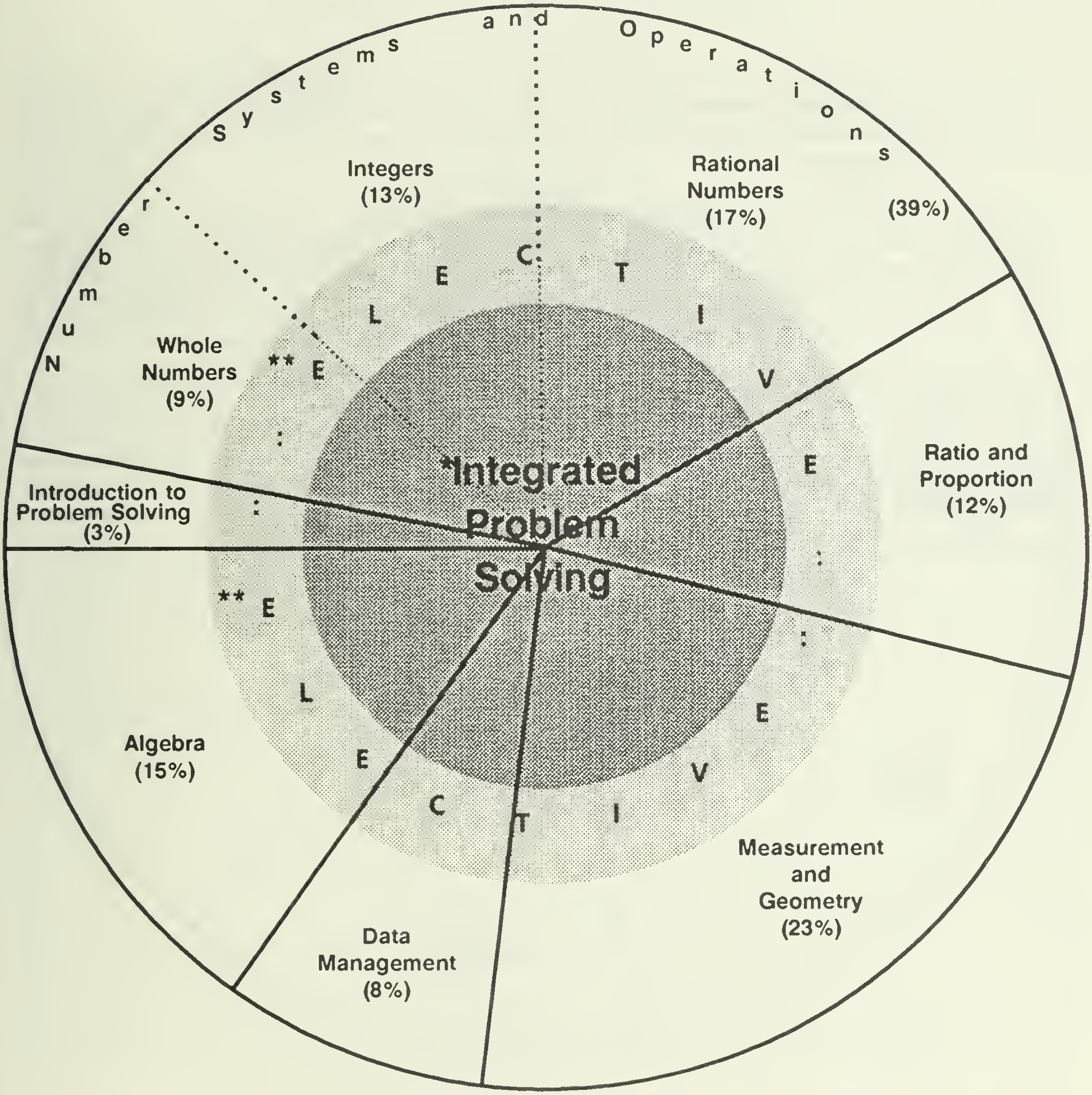
GRADE 7 MATHEMATICS



*Integrated Problem Solving - 20% of time

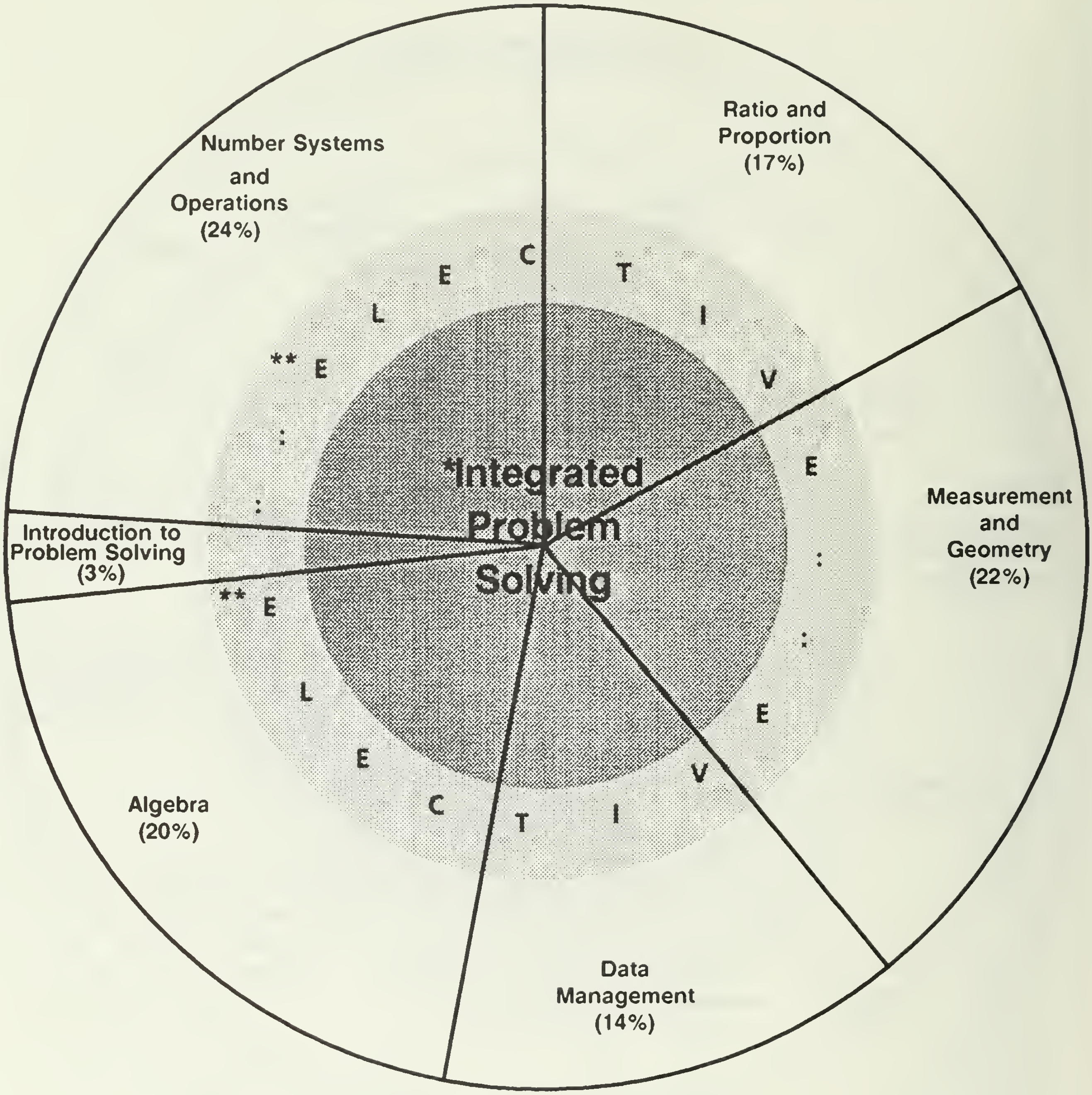
**Integrated Elective (meeting individual needs) - 20% of time

GRADE 8 MATHEMATICS



*Integrated Problem Solving - 20% of time
**Integrated Elective (meeting individual needs) - 20% of time

GRADE 9 MATHEMATICS



*Integrated Problem Solving - 20% of time
**Integrated Elective (meeting individual needs) - 20% of time

APPENDIX C

MANIPULATIVES

The use of manipulatives must not be the goal of mathematics instruction. The role of manipulatives should be to help students understand (and remember) concepts so that they are able to problem solve and apply mathematics. Once understanding has been developed, manipulatives should no longer be used.

Manipulatives should be used because of their value in learning and should not be treated as a passing fad. The emphasis for teachers, with respect to their use, should be on the **how** and not on the **what**. Explicit suggestions for use are often found in teacher resource manuals, textbooks and in professional publications such as the "Arithmetic Teacher", the "Mathematics Teacher" or "delta-K".

Manipulatives need not be expensive. Items such as dot paper, graph paper, popsicle sticks, beans, containers, glue, coloured bingo markers, rubber bands, drinking straws and pipe cleaners, used imaginatively, are as effective as commercially prepared materials. The additional benefit of using low-cost items is that all students will have access to the materials. Students will also have the opportunity to learn to use varying types of manipulatives.

GRADE 9 MATHEMATICS CURRICULUM WEIGHTING

STRAND \ TAXONOMY	Content Knowledge	Procedural Knowledge	Conceptual Under-standing	Problem Solving	Quantitative Literacy	Language of Mathematics	Technology	Psychomotor Skills	Positive Attitude	Per Cent Emphasis
Introduction to Problem Solving		○	○						○	3%
Number Systems and Operations	○	◒	●	◒	○	○	○		○	24%
Ratio and Proportion	○	○	●	◒	○	○	○		○	17%
Measurement and Geometry	○	◒	●	◒		○	○	○	○	22%
Data Management	○	○	●	○		○	○		○	14%
Algebra	○	◒	●	◒	○	○	○		○	20%
TOTAL	9%	15%	36%	19%	4%	5%	5%	1%	6%	100%

Minor Emphasis – ○

Moderate Emphasis – ◒

Major Emphasis – ●

GRADE 9 TAXONOMIC INDICATORS

It is intended that the indicators on the right provide some examples and interpretation of the taxonomic levels. They are not exhaustive.

TAXONOMIC LEVEL	INDICATORS
CONTENT KNOWLEDGE	The student can recall facts, concepts, and terminology (memorizes).
PROCEDURAL KNOWLEDGE	The student knows "how": ● performs algorithms, computations ● uses formulas ● performs constructions, conversions, order of operations ● uses calculators and computers
CONCEPTUAL UNDERSTANDING	The student knows "why", "when" and "knows that he or she knows": ● understands basic mathematics concepts ● understands relationships among: – number systems – operations – number forms (fractions, decimals, powers, etc.) – concrete, pictorial and symbolic representations ● understands place value ● understands ratio and (direct) proportion ● understands relationships within formulas ● understands relationship among geometric forms ● understands relationship among numbers and geometric forms
PROBLEM SOLVING	● understands definition of stages ● uses a variety of strategies to solve problems ● applies mathematics knowledge in unfamiliar situations

TAXONOMIC LEVEL	INDICATORS
QUANTITATIVE LITERACY	• knows basic facts • understands place value • judges the reasonableness of a solution • formulates an approximation of an outcome • knows and applies mental computational and estimation strategies • knows several computational strategies and applies each one appropriately (e.g., calculator, mental, estimation) • recognizes own computational strengths and weaknesses
LANGUAGE OF MATHEMATICS	• uses language of mathematics appropriately **without prompting**
TECHNOLOGY	• knows power and limitations of a calculator • correctly interprets the results displayed on a calculator (e.g., truncation, remainder, repeating decimals etc.) • uses a computer to solve problems and to perform iterative tasks
PSYCHOMOTOR SKILLS	• uses mathematical tools e.g., compass, protractor, ruler, calculator) and performs mathematics tasks (e.g., manipulates concrete materials)
POSITIVE ATTITUDE AND SELF-CONCEPT	• Students enjoy doing mathematics: – feel successful – persevere with tasks – enthusiastic (do more than required) – complete tasks – co-operative and helpful – physical expression (happy face, etc.) – pursue excellence • students have a positive self-concept: – willing to take risks – confident – contribute easily – feel successful

DATE DUE SLIP
